# Square Roots and Simplifying Radicals

A radical expression is an expression that contains a square root. The expression is in simplest form when the following three conditions have been met.

- No radicands have perfect square factors other than 1.
- No radicands contain fractions.
- No radicals appear in the denominator of a fraction.

The **Product Property** states that for two numbers $a$ and $b \geq 0$, $\sqrt{ab} = \sqrt{a} \cdot \sqrt{b}$.

**EXAMPLE 1**

**Simplify.**

**a.** $\sqrt{45}$

$\sqrt{45} = \sqrt{3 \cdot 3 \cdot 5}$ Prime factorization of 45

$= \sqrt{3^2} \cdot \sqrt{5}$ Product Property of Square Roots

$= 3\sqrt{5}$ Simplify.

**b.** $\sqrt{6} \cdot \sqrt{15}$

$\sqrt{6} \cdot \sqrt{15} = \sqrt{6 \cdot 15}$ Product Property

$= \sqrt{3 \cdot 2 \cdot 3 \cdot 5}$ Prime factorization

$= \sqrt{3^2} \cdot \sqrt{10}$ Product Property

$= 3\sqrt{10}$ Simplify.

For radical expressions in which the exponent of the variable inside the radical is *even* and the resulting simplified exponent is *odd*, you must use absolute value to ensure nonnegative results.

**EXAMPLE 2**

$\sqrt{20x^3y^5z^6}$

$\sqrt{20x^3y^5z^6} = \sqrt{2^2 \cdot 5 \cdot x^3 \cdot y^5 \cdot z^6}$ Prime factorization

$= \sqrt{2^2} \cdot \sqrt{5} \cdot \sqrt{x^3} \cdot \sqrt{y^5} \cdot \sqrt{z^6}$ Product Property

$= 2 \cdot \sqrt{5} \cdot x \cdot \sqrt{x} \cdot y^2 \cdot \sqrt{y} \cdot |z^3|$ Simplify.

$= 2xy^2|z^3|\sqrt{5xy}$ Simplify.

The **Quotient Property** states that for any numbers $a$ and $b$, where $a \geq 0$ and $b \geq 0$, $\sqrt{\frac{a}{b}} = \frac{\sqrt{a}}{\sqrt{b}}$.

**EXAMPLE 3**

**Simplify** $\sqrt{\frac{25}{16}}$.

$\sqrt{\frac{25}{16}} = \frac{\sqrt{25}}{\sqrt{16}}$ Quotient Property

$= \frac{5}{4}$ Simplify.

Rationalizing the denominator of a radical expression is a method used to eliminate radicals from the denominator of a fraction. To rationalize the denominator, multiply the expression by a fraction equivalent to 1 such that the resulting denominator is a perfect square.

**EXAMPLE**

**4** **Simplify.**

**a.** $\frac{2}{\sqrt{3}}$

$\frac{2}{\sqrt{3}} = \frac{2}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}}$ Multiply by $\frac{\sqrt{3}}{\sqrt{3}}$.

$= \frac{2\sqrt{3}}{3}$ Simplify.

**b.** $\frac{\sqrt{13y}}{\sqrt{18}}$

$\frac{\sqrt{13y}}{\sqrt{18}} = \frac{\sqrt{13y}}{\sqrt{2 \cdot 3 \cdot 3}}$ Prime factorization

$= \frac{\sqrt{13y}}{3\sqrt{2}}$ Product Property

$= \frac{\sqrt{13y}}{3\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}}$ Multiply by $\frac{\sqrt{2}}{\sqrt{2}}$.

$= \frac{\sqrt{26y}}{6}$ Product Property

Sometimes, conjugates are used to simplify radical expressions. Conjugates are binomials of the form $p\sqrt{q} + r\sqrt{s}$ and $p\sqrt{q} - r\sqrt{s}$.

**EXAMPLE**

**5** **Simplify** $\frac{3}{5 - \sqrt{2}}$.

$\frac{3}{5 - \sqrt{2}} = \frac{3}{5 - \sqrt{2}} \cdot \frac{5 + \sqrt{2}}{5 + \sqrt{2}}$ $\frac{5 + \sqrt{2}}{5 + \sqrt{2}} = 1$

$= \frac{3(5 + \sqrt{2})}{5^2 - (\sqrt{2})^2}$ $(a - b)(a + b) = a^2 - b^2$

$= \frac{15 + 3\sqrt{2}}{25 - 2}$ Multiply. $(\sqrt{2})^2 = 2$

$= \frac{15 + 3\sqrt{2}}{23}$ Simplify.