NAME ______________________ DATE ____________ PERIOD ____________

4-1 Study Guide and Intervention

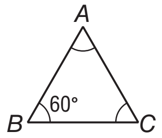
Classifying Triangles

Classify Triangles by Angles One way to classify a triangle is by the measures of its angles.

• If *all three* of the angles of a triangle are acute angles, then the triangle is an **acute triangle**.
• If *all three* angles of an acute triangle are congruent, then the triangle is an **equiangular triangle**.
• If *one* of the angles of a triangle is an obtuse angle, then the triangle is an **obtuse triangle**.
• If *one* of the angles of a triangle is a right angle, then the triangle is a **right triangle**.

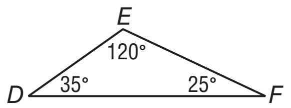
Example **Classify each triangle.**

a.



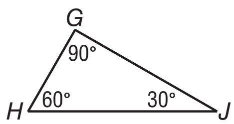
All three angles are congruent, so all three angles have measure 60°. The triangle is an equiangular triangle.

b.



The triangle has one angle that is obtuse. It is an obtuse triangle.

c.

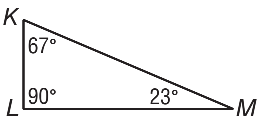


The triangle has one right angle. It is a right triangle.

Exercises

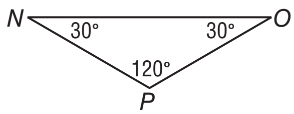
Classify each triangle as *acute*, *equiangular*, *obtuse*, or *right*.

1.



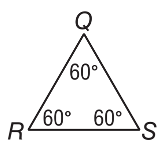
right

2.



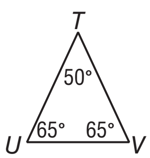
obtuse

3.



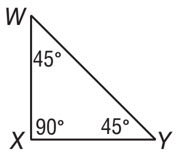
equiangular

4.



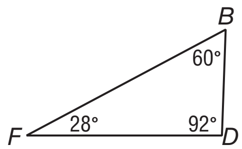
acute

5.



right

6.



obtuse

Lesson 4-1

4-1 Study Guide and Intervention *(continued)*

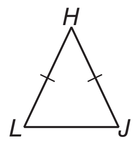
Classifying Triangles

Classify Triangles by Sides You can classify a triangle by the number of congruent sides. Equal numbers of hash marks indicate congruent sides.

- If *all three* sides of a triangle are congruent, then the triangle is an **equilateral triangle**.
- If *at least two* sides of a triangle are congruent, then the triangle is an **isosceles triangle**. Equilateral triangles can also be considered isosceles.
- If *no two* sides of a triangle are congruent, then the triangle is a **scalene triangle**.

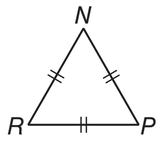
Example **Classify each triangle.**

a.



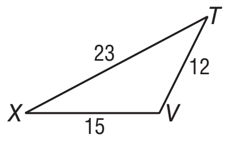
Two sides are congruent. The triangle is an isosceles triangle.

b.



All three sides are congruent. The triangle is an equilateral triangle.

c.

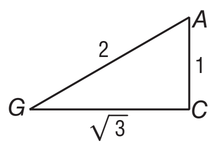


The triangle has no pair of congruent sides. It is a scalene triangle.

Exercises

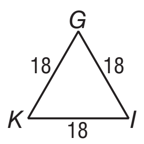
Classify each triangle as *equilateral*, *isosceles*, or *scalene*.

1.



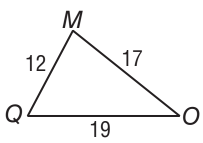
scalene

2.



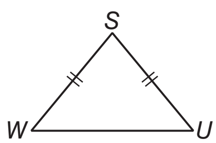
equilateral

3.



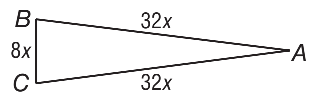
scalene

4.



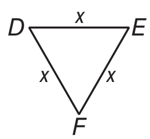
isosceles

5.



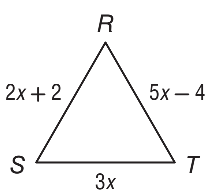
isosceles

6.



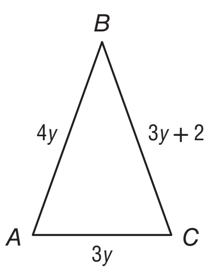
equilateral

7. ALGEBRA Find x and the length of each side if $\triangle RST$ is an equilateral triangle.



2; 6

8. ALGEBRA Find y and the length of each side if $\triangle ABC$ is isosceles with $AB = BC$.

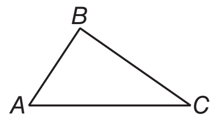


2; $AB = 8$; $BC = 8$, $AC = 6$

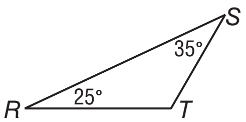
4-2 Study Guide and Intervention

Angles of Triangles

Triangle Angle-Sum Theorem If the measures of two angles of a triangle are known, the measure of the third angle can always be found.

Triangle Angle Sum Theorem	The sum of the measures of the angles of a triangle is 180. In the figure at the right, $m\angle A + m\angle B + m\angle C = 180$. 

Example 1 **Find $m\angle T$.**



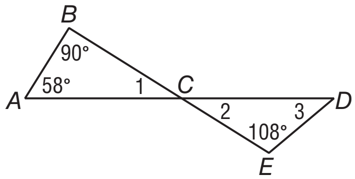
$m\angle R + m\angle S + m\angle T = 180$ Triangle Angle-Sum Theorem

$25 + 35 + m\angle T = 180$ Substitution

$60 + m\angle T = 180$ Simplify.

$m\angle T = 120$ Subtract 60 from each side.

Example 2 **Find the missing angle measures.**



$m\angle 1 + m\angle A + m\angle B = 180$ Triangle Angle-Sum Theorem

$m\angle 1 + 58 + 90 = 180$ Substitution

$m\angle 1 + 148 = 180$ Simplify.

$m\angle 1 = 32$ Subtract 148 from each side.

$m\angle 2 = 32$ Vertical angles are congruent.

$m\angle 3 + m\angle 2 + m\angle E = 180$ Triangle Angle-Sum Theorem

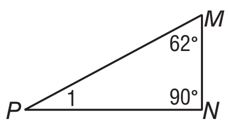
$m\angle 3 + 32 + 108 = 180$ Substitution

$m\angle 3 + 140 = 180$ Simplify.

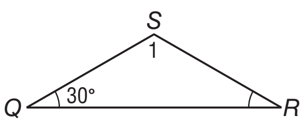
$m\angle 3 = 40$ Subtract 140 from each side.

Exercises

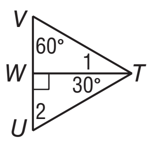
Find the measure of each numbered angle.

1. 

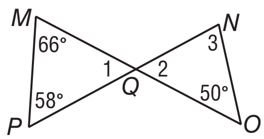
$m\angle 1 = 28$

2. 

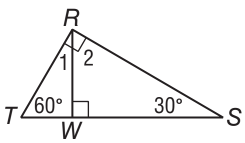
$m\angle 1 = 120$

3. 

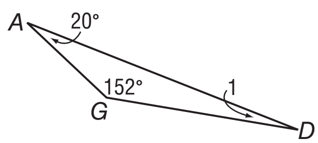
$m\angle 1 = 30$, $m\angle 2 = 60$

4. 

$m\angle 1 = 56$, $m\angle 2 = 56$, $m\angle 3 = 74$

5. 

$m\angle 1 = 30$, $m\angle 2 = 60$

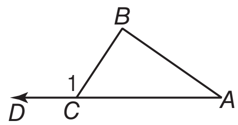
6. 

$m\angle 1 = 8$

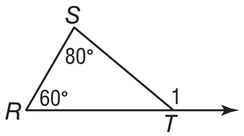
4-2 Study Guide and Intervention *(continued)*

Angles of Triangles

Exterior Angle Theorem At each vertex of a triangle, the angle formed by one side and an extension of the other side is called an **exterior angle** of the triangle. For each exterior angle of a triangle, the **remote interior angles** are the interior angles that are not adjacent to that exterior angle. In the diagram below, $\angle B$ and $\angle A$ are the remote interior angles for exterior $\angle DCB$.

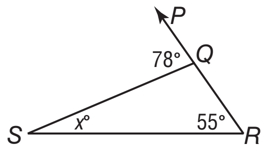
Exterior Angle Theorem	The measure of an exterior angle of a triangle is equal to the sum of the measures of the two remote interior angles. $m\angle 1 = m\angle A + m\angle B$	

Example 1 Find $m\angle 1$.



$m\angle 1 = m\angle R + m\angle S$ Exterior Angle Theorem
$= 60 + 80$ Substitution
$= 140$ Simplify.

Example 2 Find x.

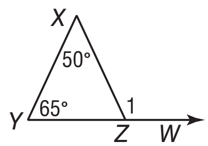


$m\angle PQS = m\angle R + m\angle S$ Exterior Angle Theorem
$78 = 55 + x$ Substitution
$23 = x$ Subtract 55 from each side.

Exercises

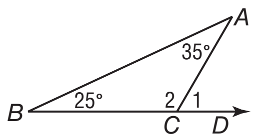
Find the measures of each numbered angle.

1.



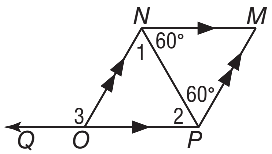
***m*∠1 = 115**

2.



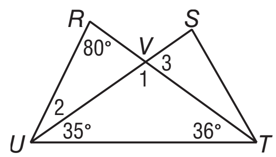
***m*∠1 = 60, *m*∠2 = 120**

3.



***m*∠1 = 60, *m*∠2 = 60, *m*∠3 = 120**

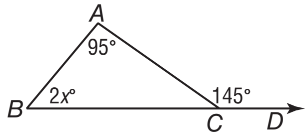
4.



***m*∠1 = 109, *m*∠2 = 29, *m*∠3 = 71**

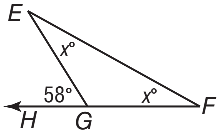
Find each measure.

5. $m\angle ABC$



50

6. $m\angle F$



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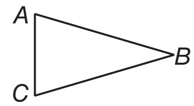
NAME ______________________________ DATE ____________ PERIOD ____________

4-6 Study Guide and Intervention

Isosceles and Equilateral Triangles

Properties of Isosceles Triangles An **isosceles triangle** has two congruent sides called the ***legs***. The angle formed by the legs is called the **vertex angle**. The other two angles are called **base angles**. You can prove a theorem and its converse about isosceles triangles.

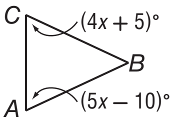


- If two sides of a triangle are congruent, then the angles opposite those sides are congruent. **(Isosceles Triangle Theorem)**
- If two angles of a triangle are congruent, then the sides opposite those angles are congruent. **(Converse of Isosceles Triangle Theorem)**

If $\overline{AB} \cong \overline{CB}$, then $\angle A \cong \angle C$.

If $\angle A \cong \angle C$, then $\overline{AB} \cong \overline{CB}$.

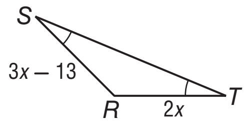
Example 1 **Find x, given $\overline{BC} \cong \overline{BA}$.**



$BC = BA$, so

$m\angle A = m\angle C$	Isos. Triangle Theorem
$5x - 10 = 4x + 5$	Substitution
$x - 10 = 5$	subtract $4x$ from each side.
$x = 15$	Add 10 to each side.

Example 2 **Find x.**

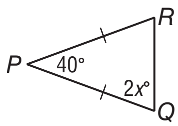


$m\angle S = m\angle T$, so

$SR = TR$	Converse of Isos. △ Thm.
$3x - 13 = 2x$	Substitution
$3x = 2x + 13$	Add 13 to each side.
$x = 13$	Subtract $2x$ from each side.

Exercises

ALGEBRA Find the value of each variable.

1. 

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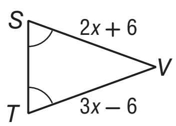
2. 

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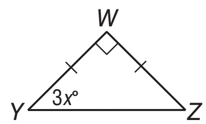
3. 

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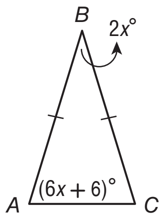
4. 

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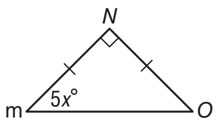
5. 

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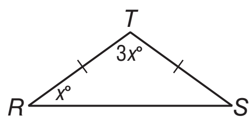
6. 

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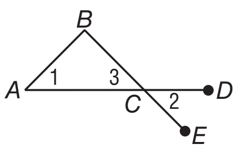
7. PROOF Write a two-column proof.

Given: $\angle 1 \cong \angle 2$

Prove: $\overline{AB} \cong \overline{CB}$



Proof:

Statements	Reasons
1. $\angle 1 \cong \angle 2$	**1. Given**
2. $\angle 2 \cong \angle 3$	**2. Vertical angles are congruent.**
3. $\angle 1 \cong \angle 3$	**3. Transitive Property of ≅**
4. $\overline{AB} \cong \overline{CB}$	**4. Converse of Isos. Triangle Thm.**

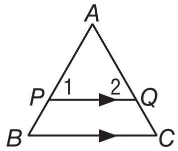
4-6 Study Guide and Intervention *(continued)*

Isosceles and Equilateral Triangles

Properties of Equilateral Triangles An **equilateral triangle** has three congruent sides. The Isosceles Triangle Theorem leads to two corollaries about equilateral triangles.

> **1.** A triangle is equilateral if and only if it is equiangular.
> **2.** Each angle of an equilateral triangle measures 60°.

Example **Prove that if a line is parallel to one side of an equilateral triangle, then it forms another equilateral triangle.**

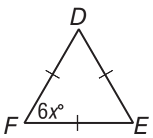


Proof:

Statements	**Reasons**
1. $\triangle ABC$ is equilateral; $\overline{PQ} \parallel \overline{BC}$.	**1.** Given
2. $m\angle A = m\angle B = m\angle C = 60$	**2.** Each $\angle$ of an equilateral $\triangle$ measures 60°.
3. $\angle 1 \cong \angle B$, $\angle 2 \cong \angle C$	**3.** If $\parallel$ lines, then corres. $\angle$s are $\cong$.
4. $m\angle 1 = 60$, $m\angle 2 = 60$	**4.** Substitution
5. $\triangle APQ$ is equilateral.	**5.** If a $\triangle$ is equiangular, then it is equilateral.

Exercises

ALGEBRA Find the value of each variable.

1.


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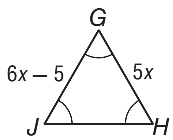
2.


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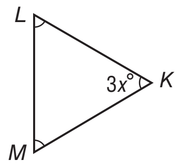
3.


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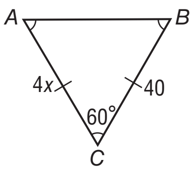
4.


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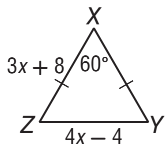
5.


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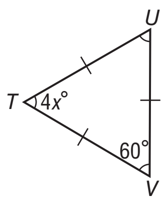
6.


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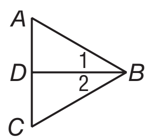
7. PROOF Write a two-column proof.

Given: $\triangle ABC$ is equilateral; $\angle 1 \cong \angle 2$.

Prove: $\angle ADB \cong \angle CDB$



Proof:

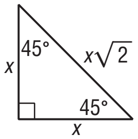
Statements	**Reasons**
1. $\triangle ABC$ is equilateral.	**1. Given**
2. $\overline{AB} \cong \overline{CB}$; $\angle A \cong \angle C$	**2. An equilateral $\triangle$ has $\cong$ sides and $\cong$ angles.**
3. $\angle 1 \cong \angle 2$	**3. Given**
4. $\triangle ABD \cong \triangle CBD$	**4. ASA Postulate**
5. $\angle ADB \cong \angle CDB$	**5. CPCTC**

8-3 Study Guide and Intervention

Special Right Triangles

Properties of 45°-45°-90° Triangles The sides of a 45°-45°-90° right triangle have a special relationship.

Example 1 **If the leg of a 45°-45°-90° right triangle is x units, show that the hypotenuse is $x\sqrt{2}$ units.**



Using the Pythagorean Theorem with $a = b = x$, then

$$c^2 = a^2 + b^2$$
$$c^2 = x^2 + x^2$$
$$c^2 = 2x^2$$
$$c = \sqrt{2x^2}$$
$$c = x\sqrt{2}$$

Example 2 **In a 45°-45°-90° right triangle the hypotenuse is $\sqrt{2}$ times the leg. If the hypotenuse is 6 units, find the length of each leg.**

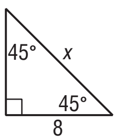
The hypotenuse is $\sqrt{2}$ times the leg, so divide the length of the hypotenuse by $\sqrt{2}$.

$$a = \frac{6}{\sqrt{2}}$$
$$= \frac{6}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}}$$
$$= \frac{6\sqrt{2}}{2}$$
$$= 3\sqrt{2} \text{ units}$$

Exercises

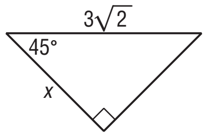
Find x.

1.



$8\sqrt{2} \approx 11.3$

2.



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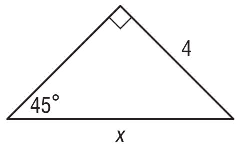
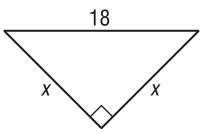
3.



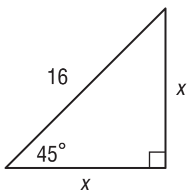
$4\sqrt{2} \approx 5.7$

4.



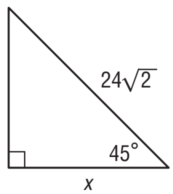
$9\sqrt{2} \approx 12.7$

5.



$8\sqrt{2} \approx 11.3$

6.



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7. If a 45°-45°-90° triangle has a hypotenuse length of 12, find the leg length.
$6\sqrt{2} \approx 8.5$

8. Determine the length of the leg of 45°-45°-90° triangle with a hypotenuse length of 25 inches. $\frac{25\sqrt{2}}{2}$ in. $\approx$ 17.7 in.

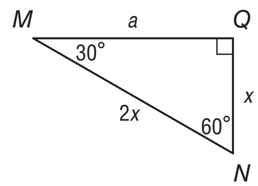
9. Find the length of the hypotenuse of a 45°-45°-90° triangle with a leg length of 14 centimeters. $14\sqrt{2}$ cm $\approx$ 19.8 cm

8-3 Study Guide and Intervention *(continued)*

Special Right Triangles

Properties of 30°-60°-90° Triangles The sides of a 30°-60°-90° right triangle also have a special relationship.

Example 1 **In a 30°-60°-90° right triangle the hypotenuse is twice the shorter leg. Show that the longer leg is $\sqrt{3}$ times the shorter leg.**



$\triangle MNQ$ is a 30°-60°-90° right triangle, and the length of the hypotenuse $\overline{MN}$ is two times the length of the shorter side $\overline{NQ}$.
Use the Pythagorean Theorem.

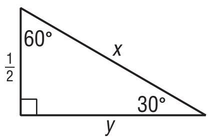
$a^2 = (2x)^2 - x^2$	$a^2 = c^2 - b^2$
$a^2 = 4x^2 - x^2$	Multiply.
$a^2 = 3x^2$	Subtract.
$a = \sqrt{3x^2}$	Take the positive square root of each side.
$a = x\sqrt{3}$	Simplify.

Example 2 **In a 30°-60°-90° right triangle, the hypotenuse is 5 centimeters. Find the lengths of the other two sides of the triangle.**

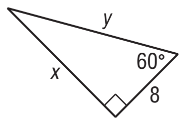
If the hypotenuse of a 30°-60°-90° right triangle is 5 centimeters, then the length of the shorter leg is one-half of 5, or 2.5 centimeters. The length of the longer leg is $\sqrt{3}$ times the length of the shorter leg, or $(2.5)(\sqrt{3})$ centimeters.

Exercises

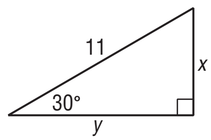
Find x and y.

1.


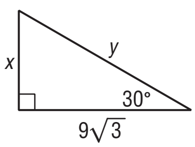
$x = 1$;
$y = 0.5\sqrt{3} \approx 0.9$

2.


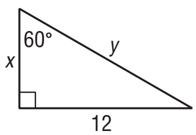
$x = 8\sqrt{3} \approx 13.9$;
$y = 16$

3.


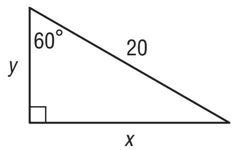
$x = 5.5$;
$y = 5.5\sqrt{3} \approx 9.5$

4.


$x = 9$;
$y = 18$

5.


$x = 4\sqrt{3} \approx 6.9$;
$y = 8\sqrt{3} \approx 13.9$

6.


$x = 10\sqrt{3} \approx 17.3$;
$y = 10$

7. An equilateral triangle has an altitude length of 36 feet. Determine the length of a side of the triangle.

$24\sqrt{3}$ feet ≈ 41.6 ft

8. Find the length of the side of an equilateral triangle that has an altitude length of 45 centimeters.

$30\sqrt{3}$ cm ≈ 52 cm