Integrated Math III Summer Assignment

Dear Da Vinci Juniors,

Welcome to your junior math course. I am excited for an amazing year of commitment and dedication to Mathematics. First off, I apologize for not being able to speak to you in person. Please read this letter in its full to understand your summer assignment and the reasoning behind it.

Integrated Math III is your first higher division mathematics course that you will take at Da Vinci Design. You will be engaging in high-level mathematics that will require you to use everything you have learned up to now to make creative solutions to solve difficult inquiry problems. Therefore, in order for you to be successful in my class, you MUST have a solid understanding of foundational mathematics.

The following packet is your summer homework. Why give summer homework? Research shows that students who do not engage in educational learning throughout the summer backtrack on their learning and start the new school year already behind. This is especially true for math. You are expected to know how to factor quadratics and solve exponents when this class starts. Unfortunately, students often lose these skills over the summer. It is my goal to help you be the best mathematicians you can be. This means that you MUST make sure you do not backtrack but instead make gains and learn during this summer. This summer packet will not only refresh on things you should know by now, it will help teach you mathematical truths that you did not know before! The packet also features examples for each problem that should help you immensely as you solve them.

As you flip through this packet you may think, what an enormous amount of work! How could a teacher expect a student to do all of this in one summer! Before you start complaining, understand this.

1. Many of the problems are extremely short. If you are where the average 10th grade should be, many of these problems should take less than 30 seconds.
2. Some of these problems, however, WILL be hard for you. Why? Because you will most likely run into a problem that you forgot how to do or have not really learned before. This is your opportunity to practice a skill you will need in my class and for the rest of your lives: how to learn a concept through your own independent effort. Looking at the examples given, you should be able to see patterns and learn from them.
3. I am giving this summer assignment because I care about your learning and I want to see you all succeed in high-level mathematics.

I understand that many students have had a negative experience with math in the past. However, I hope that you can become empowered by coming to the realization that you can do rigorous and difficult material because you are capable when you do not give up and persevere.

Here is what you need to do:

1. Get started on the packet on a separate piece of paper in pencil. You are only doing the odds.
2. Look at each example and apply its concepts to the next problems.
3. Justify each problem. If you do not know how, please refer to the examples in the next pages.
4. If you get stuck, leave a space to justify your thinking. Read the example carefully to try to learn it. Reach out to me, a tutor, an online resource, etc. if you are still struggling to make sure you learn it before the next year starts!
5. Continue until you finish at least 10 problems.
6. Do this for each weekday and you should be finished with the packet with time to spare!
7. Staple all pages together.
8. **Math Summer packet is due on the first day of school. It is worth 100 points.**

If you have any questions, please refer to the FAQ's page. If your question is not listed, then please feel free to ask me any questions by emailing jhwang@davincischools.org or coming to my room (204).

Yours Truly,

Mr. Hwang

FAQ: FREQUENTLY ASKED QUESTIONS

What happens if I don’t finish the packet by the time school starts?
Ultimately, it is your choice to finish the packet. However, if you do not finish the packet, expect a lecture on what it means to be responsible, and demonstrate perseverance and grit. Also, you will be losing a lot of points.

Is the assignment mandatory?
That depends - You always have a choice. If you choose not to do it, you will lose 100 points and start the year with an "I." Of course there will be an assessment to see how much you've mastered it, and that will affect your grade as well. I simply ask you that you make the right choice - the choice to complete the homework.

Why are we doing this packet?
Integrated Math III is a upper division course. You will need fundamental knowledge and basic math skills before you can interact with higher-level math. Also remember, the vision is for you to place in a college level course!

How do I justify?
Examples of justification are attached at the end of the packet.
In summary: Write down the question first. Show all you work (no mental math) step by step. Do not skip any steps and box your answer. If you are stuck on a problem, NEVER leave it blank. Write down the problem and explain your thought process. Think about how you would solve it and write it down. Then leave some space below so you can go back and fill it once you have mastered it.

How many questions are there?
In total, there are 577 total questions. I am only requiring the odd questions which makes it around 288. Given that there are about 45 weekdays this summer, there should be more than enough time to finish all of the problems if you do 10 problems per weekday. Given that some of these problems are as short as $\sqrt[2]{81}$, the amount of questions is misleading.

Elementary Algebra Test

Arithmatic

1) $(0.12)^2 =$

$$\begin{array}{r} 0.12 \\ \cdot\ 0.12 \\ \hline 024 \\ 012 \\ +000 \\ \hline \end{array}$$

$\boxed{.00144}$ = A

$$\begin{array}{r} 0.12 \\ \times\ 0.12 \\ \hline 024 \\ 012 \\ +000 \\ \hline \end{array}$$

$\boxed{.00144}$

Polynomials

2) One of the factors o $x^2 - x - 6$ is...

$x^2 - x - 6$

$-3 \quad 2$

$(x-3)\boxed{(x+2)} \rightarrow$ B) $x+2$

Write down ALL questions. Then proceed to showing your work
Notice how this student show each step clearly and
boxed the answer.

Linear equations and Inequalities

3) If $6x - 3 = 8x - 9$, then $x =$

$8x - 9 = 6x - 3$

$\underset{-6x}{8x} = \underset{-6x}{6x} + 6$

$\frac{2x}{2} = \frac{6}{2}$

$\boxed{x = 3}$ C

Mathematical Justification
- Show me a step-by-step process.
Mental math is not justification.
You must explicitly demonstrate each step

Quadratic Equations

4) What are the possible values of x such that $3x^2 - 2x = 0$

$3x^2 - 2x = 0$

$x(3x - 2) =$

0 or $\frac{2}{3}$ $\rightarrow$ D)

Intermediate Algebra Test

1. Elementary numeric algebraic operations:

$$\frac{c}{d} + 2 = \frac{c+2d}{d} \text{ (A)}$$

2. Rational Expression:

$\frac{c-d}{\frac{1}{d}-\frac{1}{c}}$ = Im not sure how to approach this problem, I thought I should subtract the $\frac{1}{d}-\frac{1}{c}$ but then I realized that wouldnt help nor would I know where to go after that.

If you are not sure how to solve a problem, do not skip it. Proceed to explaining your thought process. How would you start it? Where did you get stuck? What do you think you should do in this situation?

Notice how this student left spaces below so that she can go back to fill it in.

3. Exponents and Radicals

$\sqrt{3} + \sqrt{27}$ = Not sure how to approach this either, I know how to factor them both out, but im not sure if I should factor them then add those two together

4. Linear Equations:

if $3x+2y=8$ and $y=x-1$, then $x=$?

$$3x + 2x - 1 = 8$$

$$5x - 1 \overset{+1}{=} 8^{+1}$$

$$\frac{5x}{5} = \frac{9}{5} > x = \frac{9}{5} \text{ (D)}$$

Elementary Algebra Diagnostic Test Practice – Topic 1: Arithmetic Operations

Directions: Study the examples, work the problems, then check your answers on the back of this sheet. If you don't get the answer given, check your work and look for mistakes. If you have trouble, ask a math teacher or someone else who understands this topic.

A. Fractions

Simplifying Fractions:

Example: Reduce 27/36:

$$\frac{27}{36}=\frac{9\cdot 3}{9\cdot 4}=\frac{9}{9}\cdot\frac{3}{4}=1\cdot\frac{3}{4}=\frac{3}{4}$$

(Note that you must be able to find a common factor—in this case 9—in both the top and bottom in order to reduce.)

1 to 3: *Reduce*:

1. $\frac{13}{52}=$ | 2. $\frac{26}{65}=$ | 3. $\frac{3+6}{3+9}=$

Equivalent Fractions:

Example:

1) 3/4 is the equivalent to how many eighths? $\frac{3}{4}=\frac{?}{8}$

$$\frac{3}{4}=1\cdot\frac{3}{4}=\frac{2}{2}\cdot\frac{3}{4}=\frac{2\cdot 3}{2\cdot 4}=\frac{6}{8}$$

4 to 5: *Complete*:

4. $\frac{4}{9}=\frac{?}{72}$ | 5. $\frac{3}{5}=\frac{?}{20}$

How to Get the Lowest Common Denominator (LCD) by finding the least common multiple (LCM) of all denominators:

Example: 5/6 and 8/15

First find LCM of 6 and 15:

$6=2\cdot 3$ $\quad 15=3\cdot 5$

$\text{LCM}=2\cdot 3\cdot 5=30$, so

$\frac{5}{6}=\frac{25}{30}$, and $\frac{8}{15}=\frac{16}{30}$

6 to 7: *Find equivalent fractions with the LCD:*

6. $\frac{2}{3}$ and $\frac{2}{9}$ | 7. $\frac{3}{8}$ and $\frac{7}{12}$

8. Which is larger, 5/7 or 3/4? (Hint: find LCD fractions)

Adding, Subtracting Fractions: If denominators are the same, combine the numerators:

Example:

$$\frac{7}{10}-\frac{1}{10}=\frac{7-1}{10}=\frac{6}{10}=\frac{3}{5}$$

9 to 11: *Find the sum or difference (reduce if possible)*:

9. $\frac{4}{7}+\frac{2}{7}=$

10. $\frac{5}{6}+\frac{1}{6}=$

11. $\frac{7}{8}-\frac{5}{8}=$

If denominators are different, find equivalent fractions with common denominators, then proceed as before:

Examples:

1) $\frac{4}{5}+\frac{2}{3}=\frac{12}{15}+\frac{10}{15}=\frac{22}{15}=1\frac{7}{15}$

2) $\frac{1}{2}-\frac{2}{3}=\frac{3}{6}-\frac{4}{6}=\frac{3-4}{6}=\frac{-1}{6}$

12 to 13: *Simplify*:

12. $\frac{3}{5}-\frac{2}{3}=$ | 13. $\frac{5}{8}+\frac{1}{4}=$

Multiplying Fractions: multiply the tops, multiply the bottoms, reduce if possible.

Example: $\frac{3}{4}\cdot\frac{2}{5}=\frac{3\cdot 2}{4\cdot 5}=\frac{6}{20}=\frac{3}{10}$

14 to 17: *Simplify*:

14. $\frac{2}{3}\cdot\frac{3}{8}=$

15. $\frac{1}{2}\cdot\frac{1}{3}=$

16. $\left(\frac{3}{4}\right)^2=$

17. $\left(2\frac{1}{2}\right)^2=$

Dividing Fractions: a nice way to do this is to make a compound fraction and then multiply the top and bottom (of the big fraction) by the LCD of both:

Examples:

1) $\frac{3}{4}\div\frac{2}{3}=\frac{\frac{3}{4}}{\frac{2}{3}}=\frac{\frac{3}{4}\cdot 12}{\frac{2}{3}\cdot 12}=\frac{9}{8}$

2) $\frac{7}{\frac{2}{3}-\frac{1}{2}}=\frac{7\cdot 6}{\left(\frac{2}{3}-\frac{1}{2}\right)\cdot 6}=\frac{42}{4-3}=\frac{42}{1}=42$

18 to 22: *Simplify*:

18. $\frac{3}{2}\div\frac{1}{4}=$

19. $11\frac{3}{8}\div\frac{3}{4}=$

20. $\frac{3}{4}\div 2=$

21. $\frac{2}{\frac{3}{4}}=$

22. $\frac{\frac{2}{3}}{4}=$

B. Decimals

Meaning of Places: in 324.519, each digit position has a value ten times the place to its right. The part to the left of the point is the whole number part. Right of the point, the places have values: tenths, hundredths, etc., so 324.519 – (3 x 100) + (2 x 10) + (4 x 1) + (5 x 1/10) + (1 x 1/100) + (9 x 1/1000).

23. Which is larger: .59 or .7 ?

To Add or Subtract Decimals, like places must be combined (line up the points).

Examples:

1) 1.23 – 0.1 = 1.13
2) 4 + 0.3 = 4.3
3) 6.04 – (2 – 1.4) = 6.04 – 0.6 = 5.44

24 to 27: *Simplify*.

24. 5.4 + 0.78 =
25. 0.36 – 0.63 =
26. 4 – 0.3 + 0.001 – 0.01 + 0.1 =
27. \$3.54 – \$1.68 =

Multiplying Decimals

Examples:

1) .3 x .5 = .15
2) .3 x .2 = .06
3) $(.03)^2=.0009$

28 to 31: *Simplify*:

28. 3.24 x 10 =
29. .01 x .2 =
30. $(.51)^2=$
31. 5 x .4 =

Dividing Decimals: change the problem to an equivalent whole number problem by multiplying both by the same power of ten.

Examples:

1) $0.3\div 0.03$
Multiply both by 100 to get $30\div 3=10$

2) $\frac{.014}{.07}$ Multiply both by 1000, get $\frac{14}{70}=14\div 70=.2$

32 to 34: *Simplify*:

32. $0.013\div 100=$
33. $0.053\div 0.2=$
34. $\frac{340}{3.4}=$

C. Positive Integer Exponents and Square Roots of Perfect Squares

Meaning of Exponents (powers):

Examples:

1) $3^4=3\cdot 3\cdot 3\cdot 3=81$
2) $4^3=4\cdot 4\cdot 4=64$

35 to 44: *Find the value*:

35. $3^2=$
36. $(-3)^2=$
37. $-(3)^2=$
38. $-3^2=$
39. $(-2)^3=$
40. $100^2=$
41. $(2.1)^2=$
42. $(-0.1)^3=$
43. $\left(\frac{2}{3}\right)^3=$
44. $\left(-\frac{2}{3}\right)^3=$

$\sqrt{a}$ is a non-negative real number if $a\ge 0$

$\sqrt{a}=b$ means $b^2=a$, where $b\ge 0$.
Thus $\sqrt{49}=7$, because $7^2=49$.
Also, $-\sqrt{49}=-7$.

One of a series of worksheets designed to provide remedial practice. Coordinated with topics on diagnostic tests supplied to the Mathematics Diagnostic Testing Project, Gayley Center Suite 304, UCLA, 405 Hilgard Ave., Los Angeles, CA 90024.

45 to 51: *Simplify*:

45. $\sqrt{144} =$
46. $-\sqrt{144} =$
47. $\sqrt{-144} =$
48. $\sqrt{8100} =$
49. $\sqrt{1.44} =$
50. $\sqrt{.09} =$
51. $\sqrt{\frac{4}{9}} =$

D. Fraction-Decimal Conversion

Fraction to Decimal: divide the top by the bottom.

Examples: 1) $\frac{3}{4} = 3 \div 4 = .75$

2) $\frac{20}{3} = 20 \div 3 = 6.666666... = 6.\overline{6}$

3) $3\frac{2}{5} = 3 + \frac{2}{5} = 3 + (2 \div 5)$
$= 3 + 0.4 = 3.4$

52 to 55: *Write each as a decimal. If the decimal repeats, show the repeating block of digits*:

52. $\frac{5}{8} =$
53. $\frac{3}{7} =$
54. $4\frac{1}{3} =$
55. $\frac{3}{100} =$

Non-repeating Decimals to Fractions: Read the number as a fraction, write it as a fraction, reduce if possible:

Examples:
1) 0.4 = four tenths $= \frac{4}{10} = \frac{2}{5}$

2) 3.76 = three and seventy-six hundredths $= 3\frac{76}{100} = 3\frac{19}{25}$

56 to 58: *Write as a fraction*:

56. 0.01 =
57. 4.9 =
58. 1.25 =

E. Percent

Meaning of Percent: translate 'percent' as 'hundredths':

Example: 8% means 8 hundredths or .08 or $\frac{8}{100} = \frac{2}{25}$

To Change a Decimal to Percent Form: multiply by 100: move the point 2 places right and write the percent symbol (%),

Examples: 1) 0.075 = 7.5%
2) 1¼ = 1.25 = 125%

59 to 60: *Write as a percent*:

59. .3 =
60. 4 =

To Change a Percent to Decimal Form, move the point 2 places left and drop the % symbol.

Examples: 1) 8.76% = 0.0876
2) 67% = 0.67

61 to 62: *Write as a decimal*:

61. 10% =
62. 0.03% =

To Solve a Percent Problem which can be written in this form:

a % of b is c .

First identify a , b , c :

63 to 65: *If each statement were written (with the same meaning) in the form a % of b is c , identify a , b , and c:*

63. 3% of 40 is 1.2
64. 600 is 150% of 400
65. 3 out of 12 is 25%

Given *a* and *b* , change *a*% to decimal form and multiply (since 'of' can be translated 'multiply').

Given *c* and one of the others , divide c by the other (first change percent to decimal, or if answer is *a* , write it as a percent).

Examples:
1) What is 9.4% of $5000?
(a% of b is c:
9.4% of $5000 is __?__)
9.4% = 0.094
0.094 x $5000 = $470 (answer)

2) 56 problems right out of 80 is what percent?
(a% of b is c: __?__% of 80 is 56)
56 ÷ 80 = 0.7 = 70% (answer)

3) 5610 people vote in an election, which is 60% of the registered voters. How many are registered?
(a% of b is c: 60% of __?__ is 5610)
60% = 0.6
5610 ÷ 0.6 = 9350 (answer)

66 to 68: *Find the answer*:

66. 4% of 9 is what?
67. What percent of 70 is 56?
68. 15% of what is 60?

F. Estimation and Approximation

Rounding to One Significant Digit:

Examples: 1) 3.67 rounds to 4
2) 0.0449 rounds to 0.04
3) 850 rounds to either 800 or 900

69 to 71: *Round to one significant digit.*

69. 45.01
70. 1.09
71. .0083

To Estimate an Answer, it is often sufficient to round each given number to one significant digit, then compute.

Example: 0.0298 x 0.000513
Round and compute:
0.03 x 0.0005 = 0.000015
0.00015 is the estimate.

72 to 75: *Select the best approximation of the answer:*

72. 1.2346825 x 367.003246 =
(4, 40, 400, 4000, 40000)

73. 0.0042210398÷0.0190498238 =
(0.02, 0.2, 0.5, 5, 20, 50)

74. 101.7283507 + 3.141592653 =
(2, 4, 98, 105, 400)

75. $(4.36285903)^3 =$
(12, 64, 640, 5000, 12000)

Elementary Algebra Diagnostic Test Practice – Topic 2: Polynomials

Directions: Study the examples, work the problems, then check your answers on the back of this sheet. If you don't get the answer given, check your work and look for mistakes. If you have trouble, ask a math teacher or someone else who understands this topic.

A. Grouping to Simplify Polynomials

The distributive property says:
$a(b + c) = ab + ac$

Examples:
1) $3(x - y) = 3x - 3y$
   $(a = 3, b = x, c = -y)$
2) $4x + 7x = (4 + 7)x = 11x$
   $(a = x, b = 4, c = 7)$
3) $4a + 6x - 2$
   $= 2(2a + 3x - 1)$

1 to 3: *Rewrite, using the distributive property.*

1. $6(x - 3) =$
2. $4x - x =$
3. $-5(a - 1) =$

Commutative and associative properties are also used in regrouping:

Examples:
1) $3x + 7 - x = 3x - x + 7$
   $= 2x + 7$
2) $5 - x + 5 = 5 + 5 - x$
   $= 10 - x$
3) $3x + 2y - 2x + 3y$
   $= 3x - 2x + 2y + 3y$
   $= x + 5y$

4 to 9: *Simplify.*

4. $x + x =$
5. $a + b - a + b =$
6. $9x - y + 3y - 8x =$
7. $4x + 1 + x - 2 =$
8. $180 - x - 90 =$
9. $x - 2y + y - 2x =$

B. Evaluation by Substitution

Examples:
1) If $x = 3$, then $7 - 4x =$
   $7 - 4(3) = 7 - 12 = -5$
2) If $a = -7$ and $b = -1$, then
   $a^2b = (-7)^2(-1) = 49(-1)$
   $= -49$
3) If $x = -2$, then
   $3x^2 - x - 5$
   $= 3(-2)^2 - (-2) - 5$
   $= 3 \cdot 4 + 2 - 5 = 12 + 2 - 5$
   $= 9$

10 to 19: *Given* $x = -1$, $y = 3$, $z = -3$, *Find the value*:

10. $2x =$
11. $-z =$
12. $xz =$
13. $y + z =$
14. $y^2 + z^2 =$
15. $2x + 4y =$
16. $2x^2 - x - 1 =$
17. $(x + z)^2 =$
18. $x^2 + z^2 =$
19. $-x^2z =$

C. Adding and Subtracting Polynomials

Combine like terms:

Examples:
1) $(3x^2 + x + 1) - (x - 1)$
   $= 3x^2 + x + 1 - x + 1$
   $= 3x^2 + 2$
2) $(x - 1) + (x^2 + 2x - 3)$
   $= x - 1 + x^2 + 2x - 3$
   $= x^2 + 3x - 4$
3) $(x^2 + x - 1) - (6x^2 - 2x + 1)$
   $= x^2 + x - 1 - 6x^2 + 2x - 1$
   $= -5x^2 + 3x - 2$

20 to 25: *Simplify*:

20. $(x^2 + x) - (x + 1) =$
21. $(x - 3) + (5 - 2x) =$
22. $(2a^2 - a) + (a^2 + a - 1) =$
23. $(y^2 - 3y - 5) - (2y^2 - y + 5) =$
24. $(7 - x) - (x - 7) =$
25. $x^2 - (x^2 + x - 1) =$

D. Monomial Times Polynomial

Use the distributive property:

Examples:
1) $3(x - 4) = 3 \bullet x + 3(-4)$
   $= 3x + (-12) = 3x - 12$
2) $(2x + 3)a = 2ax + 3a$
3) $-4x(x^2 - 1) = -4x^3 + 4x$

26 to 32: *Simplify.*

26. $-(x - 7) =$
27. $-2(3 - a) =$
28. $x(x + 5) =$
29. $(3x - 1)7 =$
30. $a(2x - 3) =$
31. $(x^2 - 1)(-1) =$
32. $8(3a^2 + 2a - 7) =$

E. Multiplying Polynomials

Use the distributive property:

$a(b + c) = ab + ac$

Example:
$(2x + 1)(x - 4)$ is $a(b + c)$ if:
$a = (2x + 1)$, $b = x$, and $c = -4$

So, $a(b + c) = ab + ac$
$= (2x + 1)x + (2x + 1)(-1)$
$= 2x^2 + x - 8x - 4$
$= 2x^2 - 7x - 4$

Short cut to multiply above two binomials: FOIL (do mentally and write answer.

F: First times First:
   $(2x)(x) = 2x^2$
O: multiply 'Outers':
   $(2x)(-4) = -8x$
I: multiply 'Inners':
   $(1)(x) = x$
L: Last times Last
   $(1)(-4) = -4$

Add, get $2x^2 - 7x - 4$

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Examples:
1) $(x+2)(x+3) = x^2 + 5x + 6$
2) $(2x-1)(x+2)$
$= 2x^2 + 3x - 2$
3) $(x-5)(x+5) = x^2 - 25$
4) $-4(x-3) = -4x + 12$
5) $(3x-4)^2 = (3x-4)(3x-4)$
$= 9x^2 - 24x + 16$
6) $(x+3)(a-5)$
$= ax - 5x + 3a - 15$

33 to 41: *Multiply.*

33. $(x+3)^2 =$

34. $(x-3)^2 =$

35. $(x+3)(x-3) =$

36. $(2x+3)(2x-3) =$

37. $(x-4)(x-2) =$

38. $-6x(3-x) =$

39. $(x - ½)^2 =$

40. $(x-1)(x+3) =$

41. $(x^2-1)(x^2+3) =$

F. Special Products

These product patterns (examples of FOIL) should be remembered and recognized:

I. $(a+b)(a-b) = a^2 - b^2$

II. $(a+b)^2 = a^2 + 2ab + b^2$

III. $(a-b)^2 = a^2 - 2ab + b^2$

Examples:
1) $(3x-1)^2 = 9x^2 - 6x + 1$
2) $(x+5)^2 = x^2 + 10x + 25$
3) $(x+8)(x-8) = x^2 - 64$

42 to 44: *Match each pattern with its example.*

42. I:

43. II:

44. III:

45 to 52: *Write the answer using the appropriate product pattern:*

45. $(3a+1)(3a-1) =$

46. $(y-1)^2 =$

47. $(3a+2)^2 =$

48. $(3a+2)(3a-2) =$

49. $(3a-2)(3a-2) =$

50. $(x-y)^2 =$

51. $(4x+3y)^2 =$

52. $(3x+y)(3x-y) =$

G. Factoring

Monomial Factors:
$ab + ac = a(b+c)$

Examples:
1) $x^2 - x = x(x-1)$
2) $4x^2y + 6xy = 2xy(2x+3)$

Difference of Two Squares:
$a^2 - b^2 = (a+b)(a-b)$

Example:
$9x^2 - 4 = (3x+2)(3x-2)$

Trinomial Square:
$a^2 + 2ab + b^2 = (a+b)^2$
$a^2 - 2ab + b^2 = (a-b)^2$

Example:
$x^2 - 6x + 9 = (x-3)^2$

Trinomial:

Example:
1) $x^2 - x - 2 = (x-2)(x+1)$
2) $6x^2 - 7x - 3$
$= (3x+1)(2x-3)$

53 to 67: *Factor completely*:

53. $a^2 + ab =$

54. $a^3 - a^2b + ab^2 =$

55. $8x^2 - 2 =$

56. $x^2 - 10x + 25 =$

57. $-4xy + 10x^2 =$

58. $2x^2 - 3x - 5 =$

59. $x^2 - x - 6 =$

60. $x^2y - y^2x =$

61. $x^2 - 3x - 10 =$

62. $2x^2 - x =$

63. $8x^3 + 8x^2 + 2x =$

64. $9x^2 + 12x + 4 =$

65. $6x^3y^2 - 9x^4y =$

66. $1 - x - 2x^2 =$

67. $3x^2 - 10x + 3 =$

Elementary Algebra Diagnostic Test Practice – Topic 3: Linear equations and inequalities

Directions: Study the examples, work the problems, then check your answers on the back of this sheet. If you don't get the answer given, check your work and look for mistakes. If you have trouble, ask a math teacher or someone else who understands this topic.

A. Solving One Linear Equation in One Variable:

Add or subtract the same thing on each side of the equation, or multiply or divide each side by the same thing, with the goal of getting the variable alone on one side. If there are one or more fractions, it may be desirable to eliminate then by multiplying both sides by the common denominator. If the equation is a proportion, you may wish to cross-multiply.

1 to 11: *Solve*:

1. $2x = 9$
2. $3 = \frac{6x}{5}$
3. $3x + 7 = 6$
4. $\frac{x}{3} = \frac{5}{4}$
5. $5 - x = 9$
6. $x = \frac{2x}{5} + 1$
7. $4x - 6 = x$
8. $x - 4 = \frac{x}{2} + 1$
9. $6 - 4x = x$
10. $7x - 5 = 2x + 10$
11. $4x + 5 = 3 - 2x$

To solve a linear equation for one variable in terms of the other(s), do the same as above:

Examples:

1) Solve for F : $C = \frac{5}{9}(F - 32)$

Multiply by $\frac{9}{5}$: $\frac{9}{5}C = F - 32$

Add 32: $\frac{9}{5}C + 32 = F$

Thus, $F = \frac{9}{5}C + 32$

2) Solve for b : $a + b = 90$
Subtract a : $b = 90 - a$

3) Solve for x : $ax + b = c$
Subtract b : $ax = c - b$
Divide by a : $x = \frac{c - b}{a}$

12 to 19: *Solve for the indicated variable in terms of the other(s)*:

12. $a + b = 180$
$b =$
13. $2a + 2b = 180$
$b =$
14. $P = 2b + 2h$
$b =$
15. $y = 3x - 2$
$x =$
16. $y = 4 - x$
$x =$
17. $y = \frac{2}{3}x + 1$
$x =$
18. $ax + by = 0$
$x =$
19. $by - x = 0$
$y =$

B. Solution of a One-Variable Equation Reducible to a Linear Equation:

some equations which don't appear linear can be solved by using a related linear equation.

Examples:

1) $\frac{x + 1}{3x} = -1$

Multiply by 2x : $x + 1 = -3x$
Solve: $4x = -1$
$x = -\frac{1}{4}$

(Be sure to check answer in the original equation.)

2) $\frac{3x + 3}{x + 1} = 5$

Think of 5 as $\frac{5}{1}$ and cross-multiply:

$5x + 5 = 3x + 3$
$2x = -2$
$x = -1$

But $x = -1$ doesn't make the original equation true (it doesn't check), so there is <u>no</u> solution.

20 to 25: *Solve and check*:

20. $\frac{x - 1}{x + 1} = \frac{6}{7}$
21. $\frac{3x}{2x + 1} = \frac{5}{2}$
22. $\frac{3x - 2}{2x + 1} = 4$
23. $\frac{x + 3}{2x} = 2$
24. $\frac{1}{3} = \frac{x}{x + 8}$
25. $\frac{x - 2}{4 - 2x} = 3$

Example: $|3 - x| = 2$

Since the absolute value of both 2 and –2 is 2, 3 – x can be either 2 or –2. Write these two equations and solve each:

$3 - x = 2$	or	$3 - x = -2$
$-x = -1$		$-x = -5$
$x = 1$	<u>or</u>	$x = 5$

26 to 30: *Solve*:

26. $|x| = 3$
27. $|x| = -1$
28. $|x - 1| = 3$
29. $|2 - 3x| = 0$
30. $|x + 2| = 1$

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C. Solution of Linear Inequalities

<u>Rules for inequalities</u>:

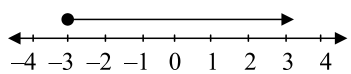
If $a > b$, then:	If $a < b$, then:
$a + c > b + c$	$a + c < b + c$
$a - c > b - c$	$a - c < b - c$
$ac > bc$ (if $c > 0$)	$ac < bc$ (if $c > 0$)
$ac < bc$ (if $c < 0$)	$ac > bc$ (if $c < 0$)
$\frac{a}{c} > \frac{b}{c}$ (if $c > 0$)	$\frac{a}{c} < \frac{b}{c}$ (if $c > 0$)
$\frac{a}{c} < \frac{b}{c}$ (if $c < 0$)	$\frac{a}{c} > \frac{b}{c}$ (if $c < 0$)

<u>Example</u>: One variable graph:
Solve and graph on a number line: $1 - 2x \leq 7$

(This is an abbreviation for: $\{x : 1 - 2x \leq 7\}$)

Subtract 1, get $-2x \leq 6$
Divide by –2, $x \geq -3$

Graph:



31 to 38: *Solve and graph on a number line*:

31. $x - 3 > 4$
32. $4x < 2$
33. $2x + 1 \leq 6$
34. $3 < x - 3$
35. $4 - 2x < 6$
36. $5 - x > x - 3$
37. $x > 1 + 4$
38. $6x + 5 \geq 4x - 3$

D. Solving a Pair of Linear Equations in Two Variables

<u>Solving a Pair of Linear Equations in Two Variables</u>: the solution consists of an ordered pair, an infinite number of ordered pairs, or no solution.

39 to 46: *Solve for the common solution(s) by substitution or linear combinations:*

39. $x + 2y = 7$
$3x - y = 28$

40. $x + y = 5$
$x - y = -3$

41. $2x - y = -9$
$x = 8$

42. $2x - y = 1$
$y = x - 5$

43. $2x - 3y = 5$
$3x + 5y = 1$

44. $4x - 1 = y$
$4x + y = 1$

45. $x + y = 3$
$x + y = 1$

46. $2x - y = 3$
$6x - 9 = 3y$

Directions: Study the examples, work the problems, then check your answers on the back of this sheet. If you don't get the answer given, check your work and look for mistakes. If you have trouble, ask a math teacher or someone else who understands this topic.

A. $ax^2 + bx + c = 0$: a quadratic equation can always be written so it looks like

$$ax^2 + bx + c = 0$$

where a, b, and c are real numbers and a is not zero.

Examples:

1) $5 - x = 3x^2$
   Add x: $5 = 3x^2 + x$
   Subtract 5: $0 = 3x^2 + x - 5$
   or $3x^2 + x - 5 = 0$

2) $x^2 = 3$
   Rewrite: $x^2 - 3 = 0$
   [Think of $x^2 + 0x - 3 = 0$]
   So: $a = 1$, $b = 0$, $c = -3$

1 to 5: *Write each of the following in the form* $ax^2 + bx + c = 0$, *and identify* a, b, c:

1. $3x + x^2 - 4 = 0$
2. $5 - x^2 = 0$
3. $x^2 = 3x - 1$
4. $x = 3x^2$
5. $81x^2 = 1$

B. Factoring

Monomial Factors: $ab + ac = a(b + c)$

Examples:
1) $x^2 - x = x(x - 1)$
2) $4x^2y + 6xy = 2xy(2x + 3)$

Difference of Two Squares: $a^2 - b^2 = (a + b)(a - b)$

Example: $9x^2 - 4 = (3x + 2)(3x - 2)$

Trinomial Square:
$a^2 + 2ab + b^2 = (a + b)^2$
$a^2 - 2ab + b^2 = (a - b)^2$

Example: $x^2 - 6x + 9 = (x - 3)^2$

Trinomial:

Examples:
1) $x^2 - x - 2 = (x + 1)(x - 2)$
2) $6x^2 - 7x - 3 = (3x + 1)(2x - 3)$

6 to 20: *Factor completely*:

6. $a^2 + ab =$
7. $a^3 - a^2b + ab^2 =$
8. $8x^2 - 2 =$
9. $x^2 - 10x + 25 =$
10. $-4xy + 10x^2 =$
11. $2x^2 - 3x - 5 =$
12. $x^2 - x - 6 =$
13. $x^2y - y^2x =$
14. $x^2 - 3x - 10 =$
15. $2x^2 - x =$
16. $2x^3 + 8x^2 + 8x =$
17. $9x^2 + 12x + 4 =$
18. $6x^3y^2 - 9x^4y =$
19. $1 - x - 2x^2 =$
20. $3x^2 - 10x + 3 =$

C. Solving Factored Quadratic Equations: the following statement is the central principle:

If $ab = 0$, then $a = 0$ *or* $b = 0$

First, identify a and b in $ab = 0$:

Example: $(3 - x)(x + 2) = 0$
Compare this with $ab = 0$
$a = (3 - x)$ and $b = (x + 2)$

21 to 24: *Identify* a *and* b *in each of the following*:

21. $3x(2x - 5) = 0$
22. $(x - 3)x = 0$
23. $(2x - 1)(x - 5) = 0$
24. $0 = (x - 1)(x + 1)$

Then, because $ab = 0$ means $a = 0$ *or* $b = 0$, we can use the factors to make two linear equations to solve:

Examples:

1) If $2x(3x - 4) = 0$,
then $(2x) = 0$ *or* $(3x - 4) = 0$
so, $x = 0$ *or* $x = \frac{4}{3}$

Thus, there are two solutions: 0 and $\frac{4}{3}$.

2) If $(3 - x)(x + 2) = 0$,
then $(3 - x) = 0$ *or* $(x + 2) = 0$
so, $x = 3$ *or* $x = -2$

3) If $(2x + 7)^2 = 0$, then $2x + 7 = 0$
so, $2x = -7$, and $x = -\frac{7}{2}$.

Note: there must be a zero on one side of the equation to solve by the factoring method.

25 to 31: *Solve*:

25. $(x + 1)(x - 1) = 0$
26. $4x(x + 4) = 0$
27. $0 = (2x - 5)x$
28. $0 = (2x + 3)(x - 1)$
29. $(x - 6)(x - 6) = 0$
30. $(2x - 3)^2 = 0$
31. $x(x + 2)(x - 3) = 0$

D. Solve Quadratic Equations by Factoring: Arrange the equation so zero is on one side (in the form $ax^2 + bx + c = 0$), factor, set each factor equal to zero, and solve the resulting linear equations.

Examples:

1) Solve: $6x^2 = 3x$ Rewrite: $6x^2 - 3x = 0$
Factor: $3x(2x - 1) = 0$
So, $3x = 0$ *or* $(2x - 1) = 0$
Thus $x = 0$ *or* $x = ½$

2) Solve: $0 = x^2 - x - 12$
Factor: $0 = (x - 4)(x + 3)$
Then $x - 4 = 0$ *or* $x + 3 = 0$
So, $x = 4$ *or* $x = -3$

32 to 43: *Solve by factoring*:

32. $x(x - 3) = 0$
33. $x^2 - 2x = 0$
34. $2x^2 = x$
35. $3x(x + 4) = 0$
36. $x^2 = 2 - x$
37. $x^2 + x = 6$
38. $0 = (x + 2)(x - 3)$
39. $(2x + 1)(3x - 2) = 0$
40. $6x^2 = x + 2$
41. $9 + x^2 = 6x$
42. $1 - x = 2x^2$
43. $x^2 - x - 6 = 0$

Another Problem Form: If a problem is stated in this form: 'One of the solutions of $ax^2 + bx + c = 0$ is d ', solve the equation as above, then verify the statement.

Example:

One of the solutions of $10x^2 - 5x = 0$ is:
A. –2 B. –½ C. ½
D. 2 E. 5

Solve $10x^2 - 5x = 0$ by factoring: $5x(2x - 1) = 0$
So, $5x = 0$ *or* $2x - 1 = 0$
Thus, $x = 0$ *or* $x = ½$
Since $x = ½$ is one solution, answer C is correct.

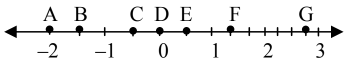
44. One of the solutions of $(x - 1)(3x + 2) = 0$ is:
A. –3/2 B. –2/3 C. 0
D. 2/3 E. 3/2

45. One solution of $x^2 - x - 2 = 0$ is:
A. –2 B. –1 C. –1/2
D. 1/2 E. 1

Directions: Study the examples, work the problems, then check your answers on the back of this sheet. If you don't get the answer given, check your work and look for mistakes. If you have trouble, ask a math teacher or someone else who understands this topic.

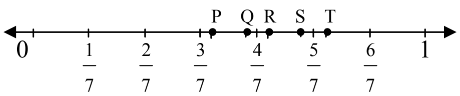
A. Graphing a Point on the Number Line

1 to 7: *Select the letter of the point on the number line with the given coordinate.*



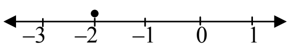
1. 0
2. ½
3. –½
4. $\frac{4}{3}$
5. –1.5
6. 2.75
7. $-\frac{3}{2}$

8 to 10: *Which letter best locates the given number*:



8. $\frac{5}{9}$
9. $\frac{3}{4}$
10. $\frac{2}{3}$

Example: $x + 3 = 1$
$x = -2$



11 to 13: *Solve each equation and graph the solution on the number line*:

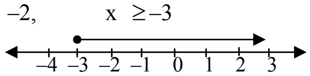
11. $2x - 6 = 0$
12. $x = 3x + 5$
13. $4 - x = 3 + x$

B. Graphing a Linear Inequality (in one variable) on the Number Line

Rules for inequalities:

If $a > b$, then:	If $a < b$, then:
$a + c > b + c$	$a + c < b + c$
$a - c > b - c$	$a - c < b - c$
$ac > bc$ (if $c > 0$)	$ac < bc$ (if $c > 0$)
$ac < bc$ (if $c < 0$)	$ac > bc$ (if $c < 0$)
$\frac{a}{c} > \frac{b}{c}$ (if $c > 0$)	$\frac{a}{c} < \frac{b}{c}$ (if $c > 0$)
$\frac{a}{c} < \frac{b}{c}$ (if $c < 0$)	$\frac{a}{c} > \frac{b}{c}$ (if $c < 0$)

Example: One variable graph:
Solve and graph on number line: $1 - 2x \le 7$
(This is an abbreviation for: $\{x: 1 - 2x \le 7\}$)
Subtract 1, get $-2x \le 6$
Divide by –2, $x \ge -3$
Graph:



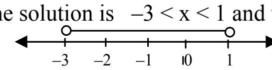
14 to 20: *Solve and graph on number line*:

14. $x - 3 > 4$
15. $4x < 2$
16. $2x + 1 \le 6$
17. $3 < x - 3$
18. $4 - 2x < 6$
19. $5 - x > x - 3$
20. $x > 1 + 4$

Examples: 1) $x > -3$ and $x < 1$
The two numbers –3 and 1 split the number line into three parts: $x < -3$, $-3 < x < 1$, and $x > 1$. Check each part to see if both $x > -3$ and $x < 1$ are true:

part	x values	x>–3 ?	x<1 ?	both true?
1	$x < -3$	no	yes	no
2	$-3 < x < 1$	yes	yes	yes (solution)
3	$x > 1$	yes	no	no

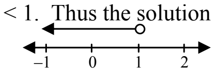
Thus the solution is $-3 < x < 1$ and the line graph is:



2) $x \le -3$ or $x < 1$ ('or' means 'and/or')

part	x values	$x \le -3$?	x<1 ?	at least one true?
1	$x \le -3$	yes	yes	yes (solution)
2	$-3 \le x < 1$	no	yes	yes (solution)
3	$x > 1$	no	no	no

So, $x \le -3$ or $-3 \le x < 1$; these cases are both covered if $x < 1$. Thus the solution is $x < 1$ and the graph is:



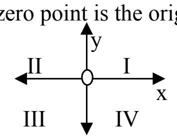
21 to 23: *Solve and graph*:

21. $x < 1$ or $x > 3$
22. $x \ge 0$ and $x > 2$
23. $x > 1$ and $x \le 4$

C. Graphing a Point in the Coordinate Plan

If two number lines intersect at right angles so that:
1) one is horizontal with positive to the right and negative to the left,
2) the other is vertical with positive up and negative down, and
3) the zero points coincide, they they form a coordinate plane, and

a) the horizontal number line is called the x-axis,
b) the vertical line is the y-axis,
c) the common zero point is the origin,
d) there are four quadrants, numbered as shown:



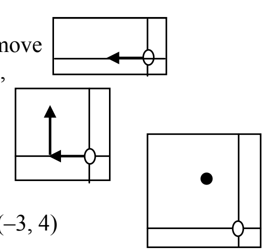
To locate a point on the plane, an ordered pair of numbers is used, written in the form (x, y). The x-coordinate is always given first.

24 to 27: *Identify x and y in each ordered pair*:

24. (3, 0)
25. (–2, 5)
26. (5, –2)
27. (0, 3)

To plot a point, start at the origin and make the two moves, first in the x-direction (horizontal) and then in the y-direction (vertical) indicated by the ordered pair.

Example: (–3, 4)
Start at the origin, move left 3 (since $x = -3$),
then (from there), up 4 (since $y = 4$).
Put a dot there to indicate the point (–3, 4)



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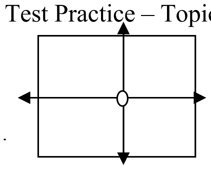
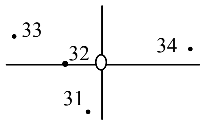
28. Join the following points in the given order: (–3, –2). (1, –4), (3, 0), (2, 3), (–1, 2), (3, 0), (–3, –2), (–1, 2), (1, –4) .



29. Two of the lines you draw cross each other. What are the coordinates of this crossing point?

30. In what quadrant does the point (a, b) lie, if $a > 0$ and $b < 0$?

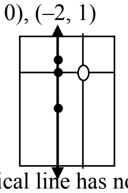
31 to 34: *For each given point, which of it coordinates, x or y , is larger?*



D. Graphing Linear Equations on the Coordinate plane: the graph of a linear equation is a line, and one way to find the line is to join points of the line. Two points determine a line, but three are often plotted on a graph to be sure they are collinear (all in a line).

Case I: If the equation looks like x = a , then there is no restriction on y, so y can be any number. Pick 3 numbers for values of y , and make 3 ordered pairs so each has x = a . Plot and join.

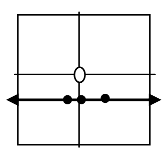
Example: x = –2
Select three *y*'s , say –3 , 0 , and 1.
Ordered pairs: (–2, –3), (–2, 0), (–2, 1)
Plot and join:



Note the slope formula gives $\frac{-3-0}{-2-(-2)} = \frac{-3}{0}$.
which is not defined: a vertical line has no slope.

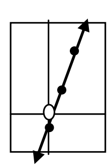
Case II: If the equation looks like y = mx + b , where either *m* or *b* (or both) can be zero, select any three numbers for values of *x* , and find the corresponding *y* values. Graph (plot) these ordered pairs and join.

Example: y = –2
Select three *x*'s , say –1 , 0 , and 2.
Since *y* must be –2, the pairs are (–1, –2) , (0, –2) , (2, –2) .



The slope formula gives $\frac{-2-(-2)}{-1-0} = \frac{0}{-1} = .0$
and the line is horizontal.

Example: y = 3x – 1



Select 3 *x* 's , say 0, 1, 2:
If x = 0 , y = 3 • 0 – 1 = –1
If x = 1 , y = 3 • 1 – 1 = 2
If x = 2 , y = 3 • 2 – 1 = 5
Ordered pairs: (0, –1), (1, 2), (2, 5)
Note the slope is $\frac{2-(-1)}{1-0} = \frac{3}{1} = 3$,
And the line is neither horizontal nor vertical.

35 to 41: *Graph each line on the number plane and find its slope (refer to section E below if necessary)*:

35. y = 3x
36. x – y = 3
37. x = 1 – y
38. y = 1
39. x = –2
40. y = –2x
41. $y = \frac{1}{2}x + 1$

E. Slope of a Line Through Two Points

42 to 47: *Find the value of each of the following*:

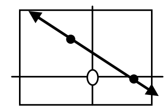
42. $\frac{3}{6} =$
43. $\frac{5-2}{1-(-1)} =$
44. $\frac{-6-(-1)}{5-10} =$
45. $\frac{0-1}{-1-4} =$
46. $\frac{0}{3} =$
47. $\frac{-2}{0} =$

The line joining the points $P_1(x_1 , y_1)$ and $P_2(x_2 , y_2)$ has slope $\frac{y_2 - y_1}{x_2 - x_1}$.

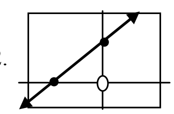
Example: A(3 , –1) , B(–2 , 4)
Slope of $\overline{AB} = \frac{4-(-1)}{-2-3} = \frac{5}{-5} = -1$

48 to 52: *Find the slope of the line joining the given points:*

48. (–3, 1) and (–1 , –4)
49. (0, 2) and (–3 , –5)
50. (3, –1) and (5 , –1)
51.



52.



Elementary Algebra Diagnostic Test Practice – Topic 6: Rational Expressions

Directions: Study the examples, work the problems, then check your answers on the back of this sheet. If you don't get the answer given, check your work and look for mistakes. If you have trouble, ask a math teacher or someone else who understands this topic.

A. Simplifying Fractional Expressions:

Example:

1) $\frac{27}{36} = \frac{9 \cdot 3}{9 \cdot 4} = \frac{9}{9} \cdot \frac{3}{4} = 1 \cdot \frac{3}{4} = \frac{3}{4}$

(note that you must be able to find a common factor—in this case 9—in both the top and bottom in order to reduce a fraction.)

2) $\frac{3a}{12ab} = \frac{3a \cdot 1}{3a \cdot 4b} = \frac{3a}{3a} \cdot \frac{1}{4b} = 1 \cdot \frac{1}{4b} = \frac{1}{4b}$

(common factor: 3a)

1 to 12: *Reduce*:

1. $\frac{13}{52} =$
2. $\frac{26}{65} =$
3. $\frac{3+6}{3+9} =$
4. $\frac{6axy}{15by} =$
5. $\frac{19a^2}{95a} =$
6. $\frac{14x - 7y}{7y} =$
7. $\frac{5a + b}{5a + c} =$
8. $\frac{x - 4}{4 - x} =$
9. $\frac{2(x+4)(x-5)}{(x-5)(x-4)} =$
10. $\frac{x^2 - 9x}{x - 9} =$
11. $\frac{8(x-1)^2}{6(x^2-1)} =$
12. $\frac{2x^2 - x - 1}{x^2 - 2x + 1} =$

Example: $\frac{3}{x} \cdot \frac{y}{15} \cdot \frac{10x}{y^2} = \frac{3 \cdot 10 \cdot x \cdot y}{15 \cdot x \cdot y^2}$
$= \frac{3}{3} \cdot \frac{5}{5} \cdot \frac{2}{1} \cdot \frac{x}{x} \cdot \frac{y}{y} \cdot \frac{1}{y}$
$= 1 \cdot 1 \cdot 2 \cdot 1 \cdot 1 \cdot \frac{1}{y} = \frac{2}{y}$

13 to 14: *Simplify*:

13. $\frac{4x}{6} \cdot \frac{xy}{y^2} \cdot \frac{3y}{2} =$
14. $\frac{x^2 - 3x}{x - 4} \cdot \frac{x(x-4)}{2x - 6} =$

B. Evaluation of Fractions

Example: If $a = -1$ and $b = 2$, find the value of $\frac{a+3}{2b-1}$.

Substitute: $\frac{-1+3}{2(2)-1} = \frac{2}{3}$

15 to 22: *Find the value, given $a = -1$, $b = 2$, $c = 0$, $x = -3$, $y = 1$, $z = 2$*:

15. $\frac{6}{b} =$
16. $\frac{x}{a} =$
17. $\frac{x}{3} =$
18. $\frac{a - y}{b} =$
19. $\frac{4x - 5y}{3y - 2x} =$
20. $\frac{b}{c} =$
21. $-\frac{b}{z} =$
22. $\frac{c}{z} =$

C. Equivalent Fractions

Examples:

1) $\frac{3}{4}$ is the equivalent to how many eighths?

$\frac{3}{4} = \frac{?}{8}$ $\qquad \frac{3}{4} = 1 \cdot \frac{3}{4} = \frac{2}{2} \cdot \frac{3}{4} = \frac{2 \cdot 3}{2 \cdot 4} = \frac{6}{8}$

2) $\frac{6}{5a} = \frac{?}{5ab}$ $\qquad \frac{6}{5a} = \frac{b}{b} \cdot \frac{6}{5a} = \frac{6b}{5ab}$

3) $\frac{3x+2}{x+1} = \frac{?}{4(x+1)}$

$\frac{3x+2}{x+1} = \frac{4}{4} \cdot \frac{3x+2}{x+1} = \frac{12x+8}{4x+4}$

4) $\frac{x-1}{x+1} = \frac{?}{(x+1)(x-2)}$

$\frac{x-1}{x+1} = \frac{(x-2)(x-1)}{(x-2)(x+1)} = \frac{x^2 - 3x + 2}{(x-2)(x+1)}$

How to get the lowest common denominator (LCD) by finding the least common multiple (LCM) of all denominators.

Examples:

1) $\frac{5}{6}$ and $\frac{8}{15}$: First find the LCM of 6 and 15:

$6 = 2 \cdot 3 \qquad 15 = 3 \cdot 5$

LCM $= 2 \cdot 3 \cdot 5 = 30$, so $\frac{5}{6} = \frac{25}{30}$, and $\frac{8}{15} = \frac{16}{30}$

2) $\frac{3}{4}$ and $\frac{1}{6a}$:

$4 = 2 \cdot 2 \qquad 6a = 2 \cdot 3 \cdot a$

LCM $= 2 \cdot 2 \cdot 3 \cdot a = 12a$, so

$\frac{3}{4} = \frac{9a}{12a}$, and $\frac{1}{6a} = \frac{2}{12a}$

3) $\frac{3}{x+2}$ and $\frac{-1}{x-2}$

LCM $= (x+2)(x-2)$, so

$\frac{3}{x+2} = \frac{3 \cdot (x-2)}{(x+2)(x-2)}$, and $\frac{-1}{x-2} = \frac{-1 \cdot (x+2)}{(x+2)(x-2)}$

23 to 27: *Complete*:

23. $\frac{4}{9} = \frac{?}{72}$
24. $\frac{3x}{7} = \frac{?}{7y}$
25. $\frac{x+3}{x+2} = \frac{?}{(x-1)(x+2)}$
26. $\frac{30 - 15a}{15 - 15b} = \frac{?}{(1+b)(1-b)}$
27. $\frac{x-6}{6-x} = \frac{?}{-2}$

28 to 33: *Find equivalent fractions with the lowest common denominator*:

28. $\frac{2}{3}$ and $\frac{2}{9}$
29. $\frac{3}{x}$ and 5
30. $\frac{x}{3}$ and $\frac{-4}{x+1}$
31. $\frac{3}{x-2}$ and $\frac{4}{2-x}$
32. $\frac{-4}{x-3}$ and $\frac{-5}{x+3}$
33. $\frac{1}{x}$ and $\frac{3x}{x+1}$

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D. Adding and Subtracting Fractions:
If denominators are the same, combine the numerators:

Example: $\frac{3x}{y}-\frac{x}{y}=\frac{3x-y}{y}=\frac{2x}{y}$

34 to 38: *Find the sum or difference as indicated (reduce if possible)*:

34. $\frac{4}{7}+\frac{2}{7}=$

35. $\frac{3}{x-3}-\frac{x}{x-3}=$

36. $\frac{b-a}{b+a}-\frac{a-b}{b+a}=$

37. $\frac{x+2}{x^2+2x}-\frac{3y^2}{xy^2}=$

38. $\frac{3a}{b}+\frac{2}{b}-\frac{a}{b}=$

If denominators are different, find equivalent fractions with common denominators, then proceed as before (combine numerators):

Examples:

1) $\frac{a}{2}-\frac{a}{4}=\frac{2a}{4}-\frac{a}{4}=\frac{2a-a}{4}=\frac{a}{4}$

2) $\frac{3}{x-1}+\frac{1}{x+2}=\frac{3(x+2)}{(x-1)(x+2)}+\frac{(x-1)}{(x-1)(x+2)}$
$=\frac{3x+6+x-1}{(x-1)(x+2)}=\frac{4x+5}{(x-1)(x+2)}$

39 to 51: *Find the sum or difference*

39. $\frac{3}{a}-\frac{1}{2a}=$

40. $\frac{3}{x}-\frac{2}{a}=$

41. $\frac{4}{5}-\frac{2}{x}=$

42. $\frac{2}{5}+2=$

43. $\frac{a}{b}-2=$

44. $a-\frac{c}{b}=$

45. $\frac{1}{a}+\frac{1}{b}=$

46. $a-\frac{1}{a}=$

47. $\frac{x}{x-1}-\frac{x}{1-x}=$

48. $\frac{3x-2}{x-2}-\frac{2}{x+2}=$

49. $\frac{2x-1}{x+1}-\frac{2x-1}{x-2}=$

50. $\frac{x}{x-2}-\frac{4}{x^2-2x}=$

51. $\frac{x}{x-2}-\frac{4}{x^2-4}=$

E. Multiplying Fractions: Multiply the tops, multiply the bottoms, reduce if possible:

Examples:

1) $\frac{3}{4}\cdot\frac{2}{5}=\frac{6}{20}=\frac{3}{10}$

2) $\frac{3(x+1)}{x-2}\cdot\frac{x^2-4}{x^2-1}=\frac{3(x+1)(x+2)(x-2)}{(x-2)(x+1)(x-1)}=\frac{3x+6}{x-1}$

52 to 59: *Multiply, reduce if possible*

52. $\frac{2}{3}\cdot\frac{3}{8}=$

53. $\frac{a}{b}\cdot\frac{c}{d}=$

54. $\frac{2}{7a}\cdot\frac{ab}{12}=$

55. $\left(\frac{3}{4}\right)^2=$

56. $\left(2\frac{1}{2}\right)^2=$

57. $\left(\frac{2a^3}{5b}\right)^3=$

58. $\frac{3(x+4)}{5y}\cdot\frac{5y^3}{x^2-16}=$

59. $\frac{x+3}{3x}\cdot\frac{x^2}{2x+6}=$

F. Dividing Fractions: A nice way to do this is to make a compound fraction and then multiply the top and bottom (of the big fraction) by the LCD of both:

Examples:

1) $\frac{a}{b}\div\frac{c}{d}=\frac{\frac{a}{b}}{\frac{c}{d}}=\frac{\frac{a}{b}\cdot bd}{\frac{c}{d}\cdot bd}=\frac{ad}{bc}$

2) $\frac{7}{\frac{2}{3}-\frac{1}{2}}=\frac{7\cdot 6}{\left(\frac{2}{3}-\frac{1}{2}\right)\cdot 6}=\frac{42}{4-3}=\frac{42}{1}=42$

3) $\frac{5x}{2y}\div 2x=\frac{\frac{5x}{2y}}{2x}=\frac{\frac{5x}{2y}\cdot 2y}{2x\cdot 2y}=\frac{5x}{4xy}=\frac{5}{4y}$

60 to 71: *Simplify*:

60. $\frac{3}{4}\Big/\frac{2}{3}=$

61. $11\frac{3}{8}\div\frac{3}{4}=$

62. $\frac{3}{4}\div 2=$

63. $\frac{a}{b}\div 3=$

64. $\frac{3}{a}\div\frac{b}{3}=$

65. $\frac{2a-b}{1/2}=$

66. $\frac{a-4}{3/a-2}=$

67. $\frac{x+7/(x^2-9)}{1/(x-3)}=$

68. $\frac{2}{3/4}=$

69. $\frac{2/3}{4}=$

70. $\frac{a/b}{c}=$

71. $\frac{a}{b/c}=$

Elementary Algebra Diagnostic Test Practice – Topic 7: Exponents and square roots

Directions: Study the examples, work the problems, then check your answers on the back of this sheet. If you don't get the answer given, check your work and look for mistakes. If you have trouble, ask a math teacher or someone else who understands this topic.

A. Positive Integer Exponents

a^b means use 'a' as a factor 'b' times. (b is the exponent or power of a .)

Examples:
1) 2^5 means $2 \cdot 2 \cdot 2 \cdot 2 \cdot 2$, and has a value of 32.
2) $c \cdot c \cdot c = c^3$

1 to 14: *Find the value.*

1. $2^3 =$
2. $3^2 =$
3. $-4^2 =$
4. $(-4)^2 =$
5. $c^4 =$
6. $1^4 =$
7. $\left(\frac{2}{3}\right)^4 =$
8. $(0.2)^2 =$
9. $\left(1\frac{1}{2}\right)^2 =$
10. $2^{10} =$
11. $(-2)^9 =$
12. $\left(2\frac{2}{3}\right)^2 =$
13. $(-1.1)^3 =$
14. $3^2 \bullet 2^3 =$

Example: Simplify:
$a \cdot a \cdot a \cdot a \cdot a = a^5$

15 to 18: *Simplify*:

15. $3^2 \cdot x^4 =$
16. $2^4 \cdot b \cdot b \cdot b =$
17. $4^2(-x)(-x)(-x) =$
18. $(-y)^4 =$

19 to 28: *Find x*:

19. $2^3 \cdot 2^4 = 2^x$
20. $\frac{2^3}{2^4} = 2^x$
21. $3^{-4} = \frac{1}{3^x}$
22. $\frac{5^2}{5^2} = 5^x$
23. $(2^3)^4 = 2^x$
24. $8 = 2^x$
25. $a^3 \cdot a = a^x$
26. $\frac{b^{10}}{b^5} = b^x$
27. $\frac{1}{c^{-4}} = c^x$
28. $\frac{a^{3y-2}}{a^{2y-3}} = a^x$

29 to 41: *Find the value*:

29. $7x^0 =$
30. $3^{-4} =$
31. $2^3 \cdot 2^4 =$
32. $0^5 =$
33. $5^0 =$
34. $(-3)^3 - 3^3 =$
35. $x^{c+3} \cdot x^{c-3} =$
36. $\frac{x^{c+3}}{x^{c-3}} =$
37. $\frac{2x^{-3}}{6x^{-4}} =$
38. $(a^{x+3})^{x-3} =$
39. $(x^3)^2 =$
40. $(3x^3)^2 =$
41. $(-2xy^2)^3 =$

B. Integer Exponents

I. $a^b \cdot a^c = a^{b+c}$

II. $\frac{a^b}{a^c} = a^{b-c}$

III. $(a^b)^c = a^{bc}$

IV. $(ab)^c = a^c \cdot b^c$

V. $\left(\frac{a}{b}\right)^c = \frac{a^c}{b^c}$

VI. $a^0 = 1$ (if $a \neq 0$)

VII. $a^{-b} = \frac{1}{a^b}$

C. Scientific Notation

Examples:
1) $32800 = 3.2800 \times 10^4$ if the zeros in the ten's and one's places are significant. If the one's zero is not, write 3.280×10^4, if neither is significant: 3.28×10^4
2) $0.004031 = 4.031 \times 10^{-3}$
3) $2 \times 10^2 = 200$
4) $9.9 \times 10^{-1} = 0.99$

Note that scientific form always looks like $a \times 10^n$ where $1 \le a < 10$, and n is an integer power of 10.

42 to 45: *Write in scientific notation*:

42. 93,000,000 =
43. 0.000042 =
44. 5.07 =
45. –32 =

46 to 48: *Write in standard notation:*

46. $1.4030 \times 10^3 =$
47. $-9.11 \times 10^{-2} =$
48. $4 \times 10^{-6} =$

To compute with numbers written in scientific form, separate the parts, compute, then recombine.

Examples:
1) $(3.14 \times 10^5)(2) =$
$(3.14)(2) \times 10^5 = 6.28 \times 10^5$
2) $\frac{4.28 \times 10^6}{2.14 \times 10^{-2}} =$
$\frac{4.28}{2.14} \times \frac{10^6}{10^{-2}} = 2.00 \times 10^8$
3) $\frac{2.01 \times 10^{-3}}{8.04 \times 10^{-6}} =$
$0.250 \times 10^3 = 2.50 \times 10^2$

49 to 56: *Write answer in scientific notation*:

49. $10^{40} \times 10^{-2} =$
50. $\frac{10^{-40}}{10^{-10}} =$
51. $\frac{1.86 \times 10^4}{3 \times 10^{-1}} =$
52. $\frac{3.6 \times 10^{-5}}{1.8 \times 10^{-8}} =$
53. $\frac{1.8 \times 10^{-8}}{3.6 \times 10^{-5}} =$
54. $(4 \times 10^{-3})^2 =$
55. $(2.5 \times 10^2)^{-1} =$
56. $\frac{(-2.92 \times 10^3)(4.1 \times 10^7)}{-8.2 \times 10^{-3}} =$

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D. Simplification of Square Roots

$\sqrt{ab} = \sqrt{a}\cdot\sqrt{b}$ if a and b are both non-negative ($a \ge 0$ and $b \ge 0$).

Examples:

1) $\sqrt{32} = \sqrt{16}\cdot\sqrt{2} = 4\sqrt{2}$

2) $\sqrt{75} = \sqrt{3}\cdot\sqrt{25} = \sqrt{3}\cdot 5 = 5\sqrt{3}$

3) If $x \ge 0$, $\sqrt{x^6} = x^3$

If $x < 0$, $\sqrt{x^6} = |x^3|$

Note: $\sqrt{a} = b$ means (by definition) that
1) $b^2 = a$, and
2) $b \ge 0$

57 to 69: *Simplify (assume all square roots are real numbers)*:

57. $\sqrt{81} =$
58. $-\sqrt{81} =$
59. $2\sqrt{9} =$
60. $4\sqrt{9} =$
61. $\sqrt{40} =$
62. $3\sqrt{12} =$
63. $\sqrt{52} =$
64. $\sqrt{\frac{9}{16}} =$
65. $\sqrt{.09} =$
66. $\sqrt{x^5} =$
67. $\sqrt{4x^6} =$
68. $\sqrt{a^2} =$
69. $\sqrt{a^3} =$

E. Adding and Subtracting Square Roots

Examples:

1) $\sqrt{5} + 2\sqrt{5} = 3\sqrt{5}$

2) $\sqrt{32} - \sqrt{2} = 4\sqrt{2} - \sqrt{2} = 3\sqrt{2}$

70 to 73: *Simplify*:

70. $\sqrt{5} + \sqrt{5} =$
71. $2\sqrt{3} + \sqrt{27} - \sqrt{75} =$
72. $3\sqrt{2} + \sqrt{2} =$
73. $5\sqrt{3} - \sqrt{3} =$

F. Multiplying Square Roots

$\sqrt{a}\cdot\sqrt{b} = \sqrt{ab}$ if $a \ge 0$ and $b \ge 0$.

Examples:

1) $\sqrt{6}\cdot\sqrt{24} = \sqrt{6\cdot 24} = \sqrt{144} = 12$

2) $\sqrt{2}\cdot\sqrt{6} = \sqrt{12} = \sqrt{4}\cdot\sqrt{3} = 2\sqrt{3}$

3) $(5\sqrt{2})(3\sqrt{2}) = 15\sqrt{4} = 15\cdot 2 = 30$

74 to 79: *Simplify*:

74. $\sqrt{3}\cdot\sqrt{3} =$
75. $\sqrt{3}\cdot\sqrt{4} =$
76. $(2\sqrt{3})(3\sqrt{2}) =$
77. $(\sqrt{9})^2 =$
78. $(\sqrt{5})^2 =$
79. $(\sqrt{3})^4 =$

80 to 81: *Find the value of x*:

80. $\sqrt{4}\cdot\sqrt{9} = \sqrt{x}$
81. $3\sqrt{2}\cdot\sqrt{5} = 3\sqrt{x}$

G. Dividing Square Roots

$\sqrt{a} \div \sqrt{b} = \frac{\sqrt{a}}{\sqrt{b}} = \sqrt{\frac{a}{b}}$, if $a \ge 0$ and $b > 0$.

Example:

$\sqrt{2} \div \sqrt{64} = \frac{\sqrt{2}}{\sqrt{64}} = \frac{\sqrt{2}}{8}$ (or $\frac{1}{8}\sqrt{2}$)

82 to 86: *Simplify*:

82. $\sqrt{3} \div \sqrt{4} =$
83. $\frac{\sqrt{9}}{\sqrt{25}} =$
84. $\frac{\sqrt{49}}{\sqrt{2}} =$
85. $\sqrt{36} \div 4 =$
86. $\frac{-8}{\sqrt{16}} =$

If a fraction has a square root on the bottom, it is sometimes desirable to find an equivalent fraction with no root on the bottom. This is called rationalizing the denominator.

Examples:

1) $\sqrt{\frac{5}{8}} = \frac{\sqrt{5}}{\sqrt{8}} = \frac{\sqrt{5}}{\sqrt{8}}\cdot\frac{\sqrt{2}}{\sqrt{2}} = \frac{\sqrt{10}}{\sqrt{16}} = \frac{\sqrt{10}}{4}$

2) $\frac{1}{\sqrt{3}} = \frac{1}{\sqrt{3}}\cdot\frac{\sqrt{3}}{\sqrt{3}} = \frac{\sqrt{3}}{\sqrt{9}} = \frac{\sqrt{3}}{9}$

87 to 94: *Simplify*:

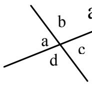
87. $\sqrt{\frac{9}{4}} =$
88. $\frac{\sqrt{18}}{\sqrt{9}} =$
89. $\frac{\sqrt{4}}{9} =$
90. $\sqrt{\frac{3}{2}} =$
91. $\frac{1}{\sqrt{5}} =$
92. $\frac{3}{\sqrt{3}} =$
93. $\frac{\sqrt{a}}{\sqrt{b}} =$
94. $\sqrt{2} + \frac{1}{\sqrt{2}} =$

Elementary Algebra Diagnostic Test Practice – Topic 8: Geometric Measurement

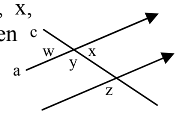
Directions: Study the examples, work the problems, then check your answers on the back of this sheet. If you don't get the answer given, check your work and look for mistakes. If you have trouble, ask a math teacher or someone else who understands this topic.

A. Intersecting lines and Parallels:

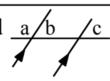
If two lines intersect as shown, adjacent angles add to 180°. *For example*, a + d = 180°. Non-adjacent angles are equal: *For example*, a = c .



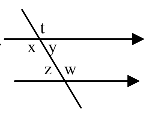
If two lines, a and b , are parallel and are cut by a third line c , forming angles w, x, y, z as shown, then x = z, w = z , w + y = 180° so z = y = 180°.



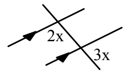
Example: If a = 3x and c = x , find the measure of c . b = c , so, b = x
a + b = 180°, so 3x + x = 180, giving 4x = 180, or x = 45.
Thus c = x = 45°.



1 to 4: *Given x = 127°* . *Find the measures of the other angles:*

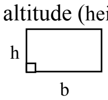


1. t
2. y
3. z
4. w
5. Find x:



B. Formulas for perimeter P and area A of triangles, squares, rectangles, and parallelograms

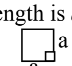
Rectangle, base b , altitude (height) h:
P = 2b + 2:
A = bh



If a wire is bent in a shape, the perimeter is the length of the wire, and the area is the number of square units enclosed by the wire.

Example:
Rectangle with b = 7 and h = 8:
P = 2b + 2h = 2 • 7 + 2 • 8
= 14 + 16 = 30 units
A = bh = 7 • 8 = 56 sq. units

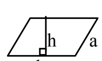
A square is a rectangle with all sides equal, so the formulas are the same (and simpler if the side length is *a*):
P = 4s
A = s^2



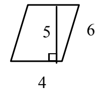
Example: Square with side 11 cm has P = 4s = 4 11 = 44 cm
A = $s^2 = 11^2 = 121$ cm^2 (sq. cm)

A parallelogram with base b and height h has A = bh

If the other side length is a , then P = 2a + 2b



Example: Parallelogram has sides 4 and 6 , and 5 is the length of the altitude perpendicular to the side 4 .

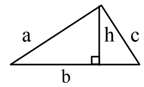


P = 2a + 2b = 2 • 6 + 2 • 4
= 12 + 8 = 20 units
A = bh = 4 • 5 = 20 sq. units

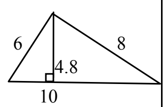
In a triangle with side lengths *a*, *b*, *c* and *h* is the altitude to side *b* ,

P = a + b + c

A = $\frac{1}{2}$bh = $\frac{\text{bh}}{2}$



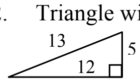
Example:
P = a + b + c
= 6 + 8 + 10
= 24 units



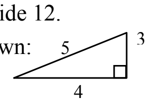
A = ½ bh = ½ (10)(4.8)
= 24 sq units

6 to 13: *Find* P *and* A *for each of the following figures*:

6. Rectangle with sides 5 and 10.
7. Rectangle, sides 1.5 and 4.
8. Square with side 3 mi.
9. Square, side $\frac{3}{4}$ yd.
10. Parallelogram with sides 36 and 24, and height 10 (on side 36).
11. Parallelogram, all sides 12, altitude 6.
12. Triangle with sides 5, 12, 13, and 5 is the height on side 12.

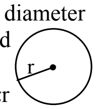


13. The triangle shown:



C. Formulas for Circle Area A and Circumference C

A circle with radius *r* (and diameter d = 2r) has distance around (circumference)
C = πd *or* C = 2πr



(If a piece of wire is bent into a circular shape, the circumference is the length of wire.)

Examples:
1) A circle with radius r = 70 has d = 2r = 140 and exact circumference C = 2πr = 2•π•70 = 140π units
2) If π is approximated by $\frac{22}{7}$. C = 140π = 140($\frac{22}{7}$) = 440 units approximately.
3) If π is approximated by 3.1, the approximate C = 140(3.1) = 434 units

The area of a circle is A = πr^2.

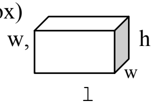
Example: If r = 8, then A = πr^2 = π •8^2 = 64π sq. units

14 to 16: *Find* C *and* A *for each circle*:

14. r = 5 units
15. r = 10 feet
16. d = 4 km

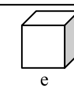
D. Formulas for Volume V

A rectangular solid (box) with length l, width w, and height *h* , has volume V = lwh .



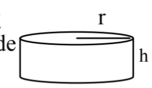
Example: A box with dimensions 3, 7 and 11 has what volume?
V = lwh = 3• 7• 11 = 231 cu. units

A cube is a box with all edges equal. If the edge is *e* , the volume V = e^3.



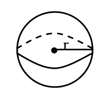
Example: A cube has edge 4 cm.
V = $e^3 = 4^3 = 64$ cm^3 (cu. cm)

A (right circular) cylinder with radius *r* and altitude *h* has V = $\pi r^2 h$.



Example: A cylinder has r = 10 and h = 14 . The exact volume is
V = $\pi r^2 h = \pi \cdot 10^2 \cdot 14$
= 1400π cu. units
If π is approximated by $\frac{22}{7}$,
V = 1400• $\frac{22}{7}$ = 4400 cu. units
If π is approximated by 3.14,
V = 1400(3.14) = 4396 cu. units

A sphere (ball) with radius r has volume
V = $\frac{4}{3}\pi r^3$



Example: The exact volume of a sphere with radius 6 in. is V = $\frac{4}{3}\pi r^3$
= $\frac{4}{3}$•π•6^3 = $\frac{4}{3}$π(216) = 288π in^3

17 to 24: *Find the exact volume of each of the following solids*:

17. Box, 6 by 8 by 9.
18. Box, $1\frac{2}{3}$ by $\frac{5}{6}$ by $2\frac{2}{5}$
19. Cube with edge 10.
20. Cube, edge 0.5
21. Cylinder with r = 5, h = 10.
22. Cylinder, r = $\sqrt{3}$, h = 2 .
23. Sphere with radius r = 2.
24. Sphere with radius r = $\frac{3}{4}$.

E. Sum of the Interior Angles of a Triangle: the three angles of any triangle add to 180°.

Example: Find the measures of angles *C* and *A* :
∠C (angle C) is marked to show its measure is 90°.
∠B + ∠C = 36 + 90 = 126, so
∠A = 180 – 126 = 54°.

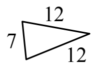
One of a series of worksheets designed to provide remedial practice. Coordinated with topics on diagnostic tests supplied to the Mathematics Diagnostic Testing Project, Gayley Center Suite 304, UCLA, 405 Hilgard Ave., Los Angeles, CA 90024.

25 to 29: *Given two angles of a triangle, find the measure of the third angle:*

25. 30^o, 60^o	
26. 115^o, 36^o	28. 82^o, 82^o
27. 90^o, 17^o	29. 68^o, 44^o

F. <u>Isosceles Triangles</u>

An isosceles triangle is defined to have at least two sides with equal measure. The equal sides may be marked: 

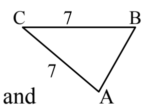
or the measures may be given: 

30 to 35: *Is the triangle isosceles?*

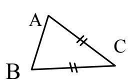
30. Sides 3, 4, 5
31. Sides 7, 4, 7
32. Sides 8, 8, 8
33. 
34. 
35. 

The angles which are opposite the equal sides also have equal measures (and all three angles add to 180^o).

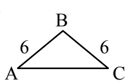
<u>Example</u>: Find the measure of $\angle A$ and $\angle C$, given $\angle B = 65^o$:
$\angle A + \angle B + \angle C = 180$, and
$\angle A = \angle B = 65$, so $\angle C = 50^o$.



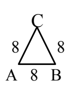
36. Find measure of $\angle A$ and $\angle B$, if $\angle C = 30^o$.



37. Find measure of $\angle B$ and $\angle C$, If $\angle A = 30^o$.



38. Find measure of $\angle A$.

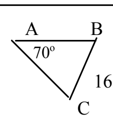


39. If the angles of a triangle are 30^o, 60^o, and 90^o, can it be isosceles?

40. If two angles of a triangle are 45^o and 60^o, can it be isosceles?

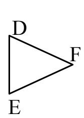
If a triangle has equal angles, the sides opposite these angles also have equal measures.

<u>Example</u>: Find the measures of $\angle B$, $\overline{AB}$ and $\overline{AC}$, given this figure, and $\angle C = 40^o$;
$\angle B = 70^o$ (because all angles add to 180^o). Since $\angle A = \angle B$,
AC = AB = 16. AB can be found with trig---later.



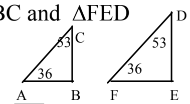
41. Can a triangle be isosceles and have a 90^o angle?

42. Given $\angle D = \angle E = 68^o$ and DF = 6. Find the measure of $\angle F$ and length of $\overline{FE}$:

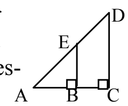


G. <u>Similar triangles</u>: If two angles of one triangle are equal to two angles of another triangle, then the triangles are similar.

<u>Example</u>: ΔABC and ΔFED are similar:
The pairs of corresponding sides are $\overline{AB}$ and $\overline{FE}$, $\overline{BC}$ and $\overline{BD}$, and $\overline{AC}$ and $\overline{FD}$.

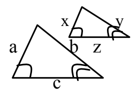


43. Name two similar triangles and list the pairs of corresponding sides.

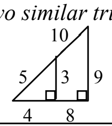


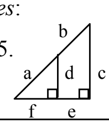
If two triangles are similar, any two corresponding sides have the same ratio (fraction value):

<u>Example</u>: The ratio a to x, or $\frac{a}{x}$, is the same as $\frac{b}{y}$ and $\frac{c}{z}$. Thus, $\frac{a}{x} = \frac{b}{y}$, $\frac{a}{x} = \frac{c}{z}$, and $\frac{b}{y} = \frac{c}{z}$. Each of these equations is called a proportion.

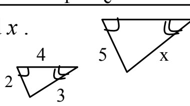


44 to 45: *Write proportions for the two similar triangles*:

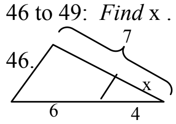
44. 

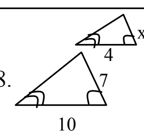
45. 

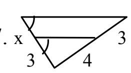
<u>Example</u>: Find x. Write and solve a proportion:
$\frac{2}{5} = \frac{3}{x}$, so $2x = 15$, $x = 7\frac{1}{2}$

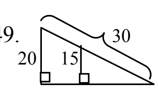


46 to 49: *Find* x.

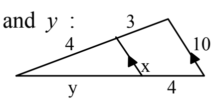
46. 

48. 

47. 

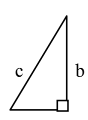
49. 

50. Find x and y:



H. <u>Pythagorean theorem</u>

In any triangle with a 90^o (right) angle, the sum of the squares of the legs equals the square of the hypotenuse. (The legs are the two shorter sides; the hypotenuse is the longest side.) If the legs have lengths a and b, and the hypotenuse length is c, then $a^2 + b^2 = c^2$ (In words, 'In a right triangle, leg squared plus leg squared equals hypotenuse squared.')

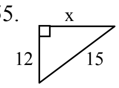


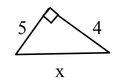
<u>Example</u>: A right triangle has hypotenuse 5 and one leg 3. Find the other leg.
Since $leg^2 + leg^2 = hypotenuse^2$,
$3^2 + x^2 = 5^2$
$9 + x^2 = 25$
$x^2 = 25 - 9 = 16$
$x^2 = \sqrt{16} = 4$

51 to 54: *Each line of the chart lists two sides of a right triangle. Find the length of the third side*:

	leg	leg	hyp
51.		15	17
52.	8		10
53.	5	12	
54.	$\sqrt{2}$	$\sqrt{3}$	

55 to 56: *Find* x.

55. 

56. 

If the sum of the squares of two sides of a triangle is the same as the square of the third side, the triangle is a right triangle.

<u>Example</u>: Is a triangle with sides 20, 29, 21 a right triangle?
$20^2 + 21^2 = 29^2$,
so it is a right triangle.

57 to 59: *Is a triangle right, if it has sides:*

57. 17, 8 15
58. 4, 5, 6
59. 60, 61, 11

Directions: Study the examples, work the problems, then check your answers on the back of this sheet. If you don't get the answer given, check your work and look for mistakes. If you have trouble, ask a math teacher or someone else who understands this topic.

A. Arithmetic, percent, and average:

1. What is the number, which when multiplied by 32, gives $32 \cdot 46$?
2. If you square a certain number, you get 9^2. What is the number?
3. What is the power of 36 that gives 36^2?
4. Find 3% of 36.
5. 55 is what percent of 88?
6. What percent of 55 is 88?
7. 45 is 80% of what number?
8. What is 8.3% of $7000?
9. If you get 36 on a 40-question test, what percent is this?
10. The 3200 people who vote in an election are 40% of the people registered to vote. How many are registered?

11 to 13: *Your wage is increased by 20%, then the new amount is cut by 20% (of the new amount).*

11. Will this result in a wage which is higher than, lower than, or the same as the original wage?
12. What percent of the original wage is this final wage?
13. If the above steps were reversed (20% cut followed by 20% increase), the final wage would be what percent of the original wage?

14 to 16: *If A is increased by 25%, it equals B.*

14. Which is larger, B or the original A ?
15. B is what percent of A?
16. A is what percent of B?
17. What is the average of 87, 36, 48, 59, and 95?
18. If two test scores are 85 and 60, what minimum score on the next test would be needed for an overall average of 80?
19. The average height of 49 people is 68 inches. What is the new average height if a 78-inch person joins the group?

B. Algebraic Substitution and Evaluation

20 to 24: *A certain TV uses 75 watts of power, and operates on 120 volts.*

20. Find how many amps of current it uses, from the relationship: volts times amps equals watts.
21. 1000 watts = 1 kilowatt (kw) . How many kilowatts does the TV use?
22. Kw times hours = kilowatt-hours (kwh). If the TV is on for six hours a day, how many kwh of electricity are used?
23. If the set is on for six hours every day of a 30-day month, how many kwh are used for the month?
24. If the electric company charges 8¢ per kwh, what amount of the month's bill is for TV power?

25 to 33: *A plane has a certain speed in still air, where it goes 1350 miles in three hours.*

25. What is its (still air) speed?
26. How far does the plane go in 5 hours?
27. How far does it go in x hours?
28. How long does it take to fly 2000 miles?
29. How long does it take to fly y miles?
30. If the plane flies against a 50 mph headwind, what is its ground speed?
31. If the plane flies against a headwind of z mph, what is its ground speed?
32. If it has fuel for 7.5 hours of flying time, how far can it go against the headwind of 50 mph.
33. If the plane has fuel for t hours of flying time, how far can it go against the headwind of z mph?

C. Ratio and proportion:

34 to 35: *x is to y as 3 is to 5.*

34. Find y when x is 7.
35. Find x when y is 7.

36 to 37: *s is proportional to P, and $P = 56$ when $s = 14$.*

36. Find s when P = 144 .
37. Find P when s = 144 .

38 to 39: *Given $3x = 4y$.*

38. Write the ratio x:y as the ratio of two integers.
39. If x = 3, find y .

40 to 41: *x and y are numbers, and two x's equal three y's.*

40. Which of x or y is the larger?
41. What is the ratio of x to y ?

42 to 44: *Half of x is the same as one-third of y .*

42. Which of x and y must be larger?
43. Write the ratio x:y as the ratio of two integers.
44. How many x's equal 30 y's?

D. Problems Leading to One Linear Equation

45. 36 is three-fourths of what number?
46. What number is ¾ of 36?
47. What fraction of 36 is 15?

48. 2/3 of 1/6 of 3/4 of a number is 12. What is the number?

49. Half the square of a number is 18. What is the number?

50. 81 is the square of twice what number?

51. Given a positive number x. Two times a positive number y is at least four times x. How small can y be?

52. Twice the square root of half of a number is 2x. What is the number?

53 to 55: *A gathering has twice as many women as men. W is the number of women and M is the number of men.*

53. Which Is correct: 2M = W or M = 2W?

54. If there are 12 women, how many men are there?

55. If the total number of men and women present is 54, how many of each are there?

56. $12,000 is divided into equal shares. Babs gets four shares, Bill gets three shares, and Ben gets the one remaining share. What is the value of one share?

E. <u>Problems Leading to Two Linear Equations</u>

57. Two science fiction coins have values x and y. Three x's and five y's have a value of 75¢, and one x and two y's have a value of 27¢. What is the value of each?

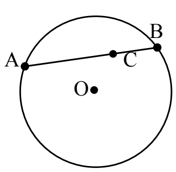
58. In mixing x gm of 3% and y gm of 8% solutions to get 10 gm of 5% solution, these equations are used:
$0.03x + 0.08y = 0.05(10)$, and $x + y = 10$.
How many gm of 3% solution are needed?

F. <u>Geometry</u>

59. Point X is on each of two given intersecting lines. How many such points X are there?

60. On the number line, points P and Q are two units apart. Q has coordinate x. What are the possible coordinates of P?

61 to 62:



61. If the length of chord AB is X and the length of CB is 16, what is AC?

62. If AC = y and CB = z, how long is AB (in terms of y and z)?

63 to 64: *The base of a rectangle is three times the height.*

63 Find the height if the base is 20.

64. Find the perimeter and area.

65. In order to construct a square with an area which is 100 times the area of a given square, how long a side should be use?

66 to 67: *The length of a rectangle is increased by 25% and its width is decreased by 40%.*

66. Its new area is what percent of its old area?

67. By what percent has the old area increased or decreased?

68. The length of a rectangle is twice the width. If both dimensions are increased by 2 cm, the resulting rectangle has 84 cm^2 more area. What was the original width?

69. After a rectangular piece of knitted fabric shrinks in length one cm and stretches in width 2 cm, it is a square. If the original area was 40 cm^2, what is the square area?

70. This square is cut into two smaller squares and two non-square rectangles as shown. Before being cut, the large square had area $(a + b)^2$. The two smaller squares have areas a^2 and b^2. Find the total area of the two non-square rectangles. Show that the areas of the 4 parts add up to the area of the original square.