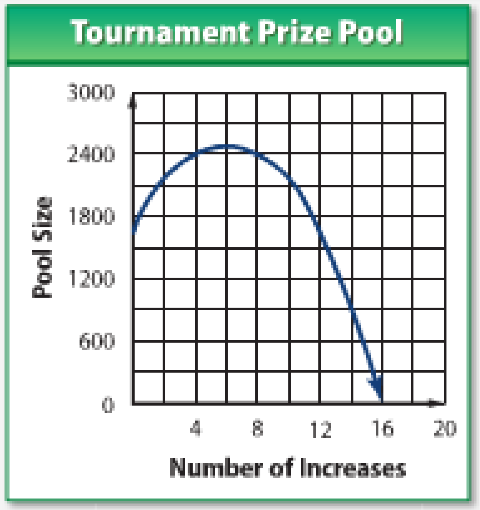
LT 3.1-3.3 Graphing Quadratic Function using the equation of the axis of symmetry

Eddie is organizing a charity tournament. He plans to charge a $20 entry fee for each of the 80 players. He recently decided to raise the entry fee by $5, and 5 fewer players entered with the increase. He used this information to determine how many fee increases will maximize the money raised.

The quadratic function at the right represents this situation. The tournament prize pool increases when he first increases the fee, but eventually the pool starts to decrease as the fee gets even higher.



Real-World Example 4 Quadratic Equations in the Real World

CHARITY **Refer to the beginning of the lesson.**

a. How much should Eddie charge in order to maximize charity income?

Words	Total	equals	fee	times	number of entrants.	
Variable	Let x = the number of price increases. Let $P(x)$ = the total pool as a function of x.					
Equation	$P(x)$	$=$	$20 + 5x$	$\cdot$	$(80 - 5x)$	

Solve for the x-value of the vertex.

$$
\begin{aligned}
P(x) &= (20 + 5x) \cdot (80 - 5x) \\
&= 20(80) + 20(-5x) + 5x(80) + 5x(-5x) && \text{Distribute.} \\
&= 1600 - 100x + 400x - 25x^2 && \text{Multiply.} \\
&= 1600 + 300x - 25x^2 && \text{Simplify.} \\
&= -25x^2 + 300x + 1600 && ax^2 + bx + c \text{ form}
\end{aligned}
$$

Use the formula for the axis of symmetry, $x = -\frac{b}{2a}$, to find the x-coordinate.

$$x = -\frac{300}{2(-25)} \text{ or } 6 \qquad a = -25 \text{ and } b = 300$$

Eddie needs to have 6 price increases, so he should charge $20 + 6(5)$ or $50.

b. What will be the maximum value of the pool?

Find the maximum value of the quadratic function $P(x)$ by evaluating $P(6)$.

$P(x) = -25x^2 + 300x + 1600$	Total pool function
$P(6) = -25(6)^2 + 300(6) + 1600$	$x = 6$
$= -900 + 1800 + 1600$ or 2500	Simplify.

Thus, the maximum prize pool is \$2500 after 6 price increases.

CHECK Graph the function on a graphing calculator and use the **CALC:maximum** function to confirm the solution.

Select a left bound of 0 and a right bound of 10. The calculator will display the coordinates of the maximum at the bottom of the screen.

[0, 10] scl: 1 by [0, 2500] scl: 100

The domain is $\{x \mid x \geq 0\}$ because there can be no negative increases in price. The range is $\{y \mid 0 \leq y \leq 2500\}$ because the prize pool cannot have a negative monetary value.

Problem #1

ECONOMICS A souvenir shop sells about 200 coffee mugs each month for \$6 each. The shop owner estimates that for each \$0.50 increase in the price, he will sell about 10 fewer coffee mugs per month.

a. How much should the owner charge for each mug in order to maximize the monthly income from their sales?

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b. What is the maximum monthly income the owner can expect to make from the mugs?

Problem #2: Maximum and Minimum

BUSINESS A store rents 1400 videos per week at $2.25 per video. The owner estimates that they will rent 100 fewer videos for each $0.25 increase in price. What price will maximize the income of the store?

Problem #3

CCSS **MODELING** A financial analyst determined that the cost, in thousands of dollars, of producing bicycle frames is $C = 0.000025f^2 - 0.04f + 40$, where f is the number of frames produced.

a. Find the number of frames that minimizes cost.

b. What is the total cost for that number of frames?

Problem #4

FINANCIAL LITERACY A babysitting club sits for 50 different families. They would like to increase their current rate of \$9.50 per hour. After surveying the families, the club finds that the number of families will decrease by about 2 for each \$0.50 increase in the hourly rate.

a. Write a quadratic function that models this situation.

b. State the domain and range of this function as it applies to the situation.

c. What hourly rate will maximize the club's income? Is this reasonable?

d. What is the maximum income the club can expect to make?

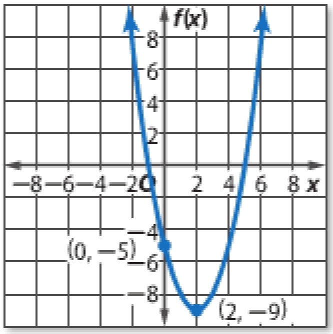
Problem #5

ACTIVITIES Last year, 300 people attended the Franklin High School Drama Club's winter play. The ticket price was \$8. The advisor estimates that 20 fewer people would attend for each \$1 increase in ticket price.

a. What ticket price would give the greatest income for the Drama Club?

b. If the Drama Club raised its tickets to this price, how much income should it expect to bring in?

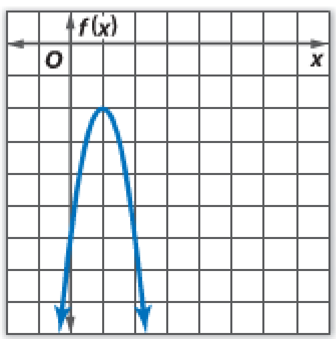
Problem #6: Determine the function represented by each graph



Higher Order Thinking

Problem #1

CRITIQUE Trent thinks that the function $f(x)$ graphed below, and the function $g(x)$ described next to it have the same maximum. Madison thinks that $g(x)$ has a greater maximum. Is either of them correct? Explain your reasoning.



$g(x)$ is a quadratic function with roots of 4 and 2 and a y-intercept of -8.

Problem #2

REASONING Determine whether the following statement is *sometimes*, *always*, or *never* true. Explain your reasoning.

In a quadratic function, if two x-coordinates are equidistant from the axis of symmetry, then they will have the same y-coordinate.