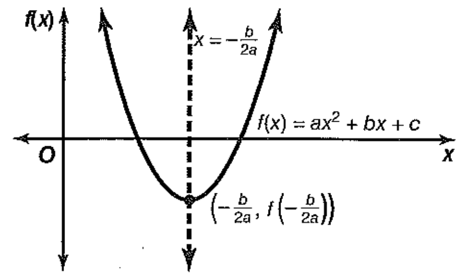
Enrichment

3-1

Finding the x-intercepts of a Parabola

As you know, if $f(x) = ax^2 + bx + c$ is a quadratic function, the values of x that make $f(x)$ equal to zero are $\frac{-b + \sqrt{b^2 - 4ac}}{2a}$ and $\frac{-b - \sqrt{b^2 - 4ac}}{2a}$.

The average of these two number values is $-\frac{b}{2a}$. The function $f(x)$ has its maximum or minimum value when $x = -\frac{b}{2a}$. The x-intercepts of the parabola, when they exist, are $\frac{\sqrt{b^2 - 4ac}}{2a}$ units to the left and right of the axis of symmetry.



Example **Find the vertex, axis of symmetry, and x-intercepts for $f(x) = 5x^2 + 10x - 7$.**

Use $x = -\frac{b}{2a}$.

$x = -\frac{10}{2(5)} = -1$ The x-coordinate of the vertex is -1.

Substitute $x = -1$ in $f(x) = 5x^2 + 10x - 7$.

$f(-1) = 5(-1)^2 + 10(-1) - 7 = -12$. The vertex is $(-1, -12)$.

The axis of symmetry is $x = -\frac{b}{2a}$, or $x = -1$.

The x-coordinates of the x-intercepts are $-1 \pm \frac{\sqrt{b^2 - 4ac}}{2a} = -1 \pm \frac{\sqrt{10^2 - 4 \cdot 5 \cdot (-7)}}{2 \cdot 5}$ $= -1 \pm \frac{\sqrt{240}}{10}$. The x–intercepts are $\left(-1 - \frac{2}{5}\sqrt{15}, 0\right)$ and $\left(-1 + \frac{2}{5}\sqrt{15}, 0\right)$.

Exercises

Find the vertex, axis of symmetry, and x-intercepts for the graph of each function using $x = -\frac{b}{2a}$.

1. $f(x) = x^2 - 4x - 8$ **$(2, -12)$; $x = 2$; $2 \pm 2\sqrt{3}$**

2. $g(x) = -4x^2 - 8x + 3$ **$(-1, 7)$; $x = -1$; $-1 \pm 2\sqrt{7}$**

3. $y = -x^2 + 8x + 3$ $(-1)(x)^2$ **$(4, 19)$; $x = 4$; $4 \pm \sqrt{19}$**

4. $f(x) = 2x^2 + 6x + 5$ **$\left(-\frac{3}{2}, \frac{1}{2}\right)$; $x = -\frac{3}{2}$; no x-intercepts**

5. $A(x) = x^2 + 12x + 36$ **$(-6, 0)$; $x = -6$; -6**

6. $k(x) = -2x^2 + 2x - 6$ **$\left(\frac{1}{2}, -5\frac{1}{2}\right)$; $x = \frac{1}{2}$; no x-intercepts**