

4.1 Complex Numbers (Notes)

Simplify

NA

EX 1: $\sqrt{-27}$

$$\sqrt{-1} \cdot \sqrt{27}$$

$$i \cdot \sqrt{9 \cdot 3}$$

$$3i\sqrt{3}$$

$\sqrt{-216}$

$$\sqrt{-1} \cdot \sqrt{216}$$

$$i \cdot \sqrt{36 \cdot 6}$$

$$6i\sqrt{6}$$

EX 2: $\sqrt{-18}$

$$\sqrt{-1} \cdot \sqrt{18}$$

$$i \cdot \sqrt{9 \cdot 2}$$

$$3i\sqrt{2}$$

$\sqrt{-125}$

$$\sqrt{-1} \cdot \sqrt{125}$$

$$i \cdot \sqrt{25 \cdot 5}$$

$$5i\sqrt{5}$$

EX 3: $-5i \cdot 3i$

$$-15i^2$$

$$-15(-1)$$

$$15$$

$\sqrt{-6} \cdot \sqrt{-15}$

$$\sqrt{-1} \sqrt{3 \cdot 2} \cdot \sqrt{-1} \sqrt{3 \cdot 5}$$

$$\sqrt{-1} \cdot \sqrt{-1} \sqrt{3 \cdot 2 \cdot 3 \cdot 5}$$

$$i \cdot i \cdot 3\sqrt{10}$$

$$3i^2 \sqrt{10}$$

$$3(-1) \sqrt{10}$$

$$-3\sqrt{10}$$

EX 4: $3i \cdot 4i$

$$12i^2$$

$$12(-1)$$

$$-12$$

$\sqrt{20} \cdot \sqrt{-12}$

$$\sqrt{-1} \sqrt{20} \sqrt{-1} \sqrt{12}$$

$$i \cdot i \sqrt{20 \cdot 12}$$

$$i^2 \sqrt{4 \cdot 5 \cdot 4 \cdot 3}$$

$$-1 \cdot 4\sqrt{15}$$

$$-4\sqrt{15}$$

i^{31}

$$i \cdot i^{30}$$

$$i(i^2)^{15}$$

$$i(-1)^{15}$$

$$-i$$

Adding Complex Numbers

Simplify

EX 5: $\frac{4+i}{5i} \cdot \frac{5i}{5i} = \frac{20i+5i^2}{25i^2} = \frac{20i-5}{-25} = \frac{1}{5} - \frac{4}{5}i$

can't have an i in denominator

$$\frac{4+i}{5i} \cdot \frac{i}{i} = \frac{4i+i^2}{5i^2} = \frac{4i-1}{-5} = \frac{1}{5} - \frac{4}{5}i$$

Complex conjugates

$a+bi$ and $a-bi \Rightarrow$ Product is always a real number

$$\begin{aligned} \text{Ex 6: } \frac{2i}{3+6i} \cdot \frac{3-6i}{3-6i} &= \frac{6i-12i^2}{9-18i+18i-36i^2} = \frac{6i+12}{9+36} = \frac{12}{45} + \frac{6}{45}i \\ &= \frac{4}{15} + \frac{2}{15}i \end{aligned}$$

$$\begin{aligned} \text{Ex 7: } \frac{-2i}{3+5i} \cdot \frac{3-5i}{3-5i} &= \frac{-6i+10i^2}{9-15i+15i+25i^2} = \frac{-6i-10}{9+25} = \frac{-10}{34} - \frac{6}{34}i \\ &= \frac{-5}{17} - \frac{3}{17}i \end{aligned}$$

$$\text{Ex 8: } \frac{2+i}{1-i} = \frac{1+i}{1+i} = \frac{2+2i+i+i^2}{1+i-i-i^2} = \frac{2+3i-1}{1+1} = \frac{1+3i}{2} = \frac{1}{2} + \frac{3}{2}i$$

Ex 9: