NAME ____ DATE ___________________________ PERIOD ____________

LT 4.1 Enrichment #1

Conjugates and Absolute Value

When studying complex numbers, it is often convenient to represent a complex number by a single variable. For example, we might let $z = x + y\boldsymbol{i}$. We denote the conjugate of z by $\bar{z}$. Thus, $\bar{z} = x - y\boldsymbol{i}$.

We can define the absolute value of a complex number as follows.

$|z| = |x + y\boldsymbol{i}| = \sqrt{x^2 + y^2}$

There are many important relationships involving conjugates and absolute values of complex numbers.

Example 1: Show $|z|^2 = z\bar{z}$ for any complex number z.

Let $z = x + y\boldsymbol{i}$. Then,

$$
\begin{aligned}
z\bar{z} &= (x + y\boldsymbol{i})(x - y\boldsymbol{i}) \\
&= x^2 + y^2 \\
&= \sqrt{(x^2 + y^2)^2} \\
&= |z|^2
\end{aligned}
$$

Example 2: Show $\frac{\bar{z}}{|z|^2}$ is the multiplicative inverse for any nonzero complex number z.

We know $|z|^2 = z\bar{z}$. If $z \neq 0$, then we have $z\left(\frac{\bar{z}}{|z|^2}\right) = 1$.

Thus, $\frac{\bar{z}}{|z|^2}$ is the multiplicative inverse of z.

Exercises

For each of the following complex numbers, find the multiplicative inverse and prove it.

1. $2\boldsymbol{i}$

2. $-4 - 3\boldsymbol{i}$

3. $12 - 5\boldsymbol{i}$

4. $5 - 12\boldsymbol{i}$

5. $1 + i$

6. $\sqrt{3} - \boldsymbol{i}$

7. $\frac{\sqrt{3}}{3} + \frac{\sqrt{3}}{3}\boldsymbol{i}$

8. $\frac{\sqrt{2}}{2} - \frac{\sqrt{2}}{2}\boldsymbol{i}$

9. $\frac{1}{2} - \frac{\sqrt{3}}{2}\boldsymbol{i}$