4-4 Enrichment

Conjugates and Absolute Value

When studying complex numbers, it is often convenient to represent a complex number by a single variable. For example, we might let $z = x + y\boldsymbol{i}$. We denote the conjugate of z by $\bar{z}$. Thus, $\bar{z} = x - y\boldsymbol{i}$.

We can define the absolute value of a complex number as follows.

$$|z| = |x + y\boldsymbol{i}| = \sqrt{x^2 + y^2}$$

There are many important relationships involving conjugates and absolute values of complex numbers.

Example 1 **Show $|z|^2 = z\bar{z}$ for any complex number z.**

Let $z = x + y\boldsymbol{i}$. Then,

$$\begin{aligned} z\bar{z} &= (x + y\boldsymbol{i})(x - y\boldsymbol{i}) \\ &= x^2 + y^2 \\ &= \sqrt{(x^2 + y^2)^2} \\ &= |z|^2 \end{aligned}$$

Example 2 **Show $\frac{\bar{z}}{|z|^2}$ is the multiplicative inverse for any nonzero complex number z.**

We know $|z|^2 = z\bar{z}$. If $z \neq 0$, then we have $z\left(\frac{\bar{z}}{|z|^2}\right) = 1$.

Thus, $\frac{\bar{z}}{|z|^2}$ is the multiplicative inverse of z.

Exercises

For each of the following complex numbers, find the absolute value and multiplicative inverse.

1. $2\boldsymbol{i}$ **$2; \frac{-i}{2}$**

2. $-4 - 3\boldsymbol{i}$ **$5; \frac{-4 + 3i}{25}$**

3. $12 - 5\boldsymbol{i}$ **$13; \frac{12 + 5i}{169}$**

4. $5 - 12\boldsymbol{i}$ **$13; \frac{5 + 12i}{169}$**

5. $1 + \boldsymbol{i}$ **$\sqrt{2}; \frac{1 - i}{2}$**

6. $\sqrt{3} - \boldsymbol{i}$ **$2; \frac{\sqrt{3} + i}{4}$**

7. $\frac{\sqrt{3}}{3} + \frac{\sqrt{3}}{3}\boldsymbol{i}$ **$\frac{\sqrt{6}}{3}; \frac{\sqrt{3} - i\sqrt{3}}{2}$**

8. $\frac{\sqrt{2}}{2} - \frac{\sqrt{2}}{2}\boldsymbol{i}$ **$1; \frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2}i$**

9. $\frac{1}{2} - \frac{\sqrt{3}}{2}\boldsymbol{i}$ **$1; \frac{1}{2} + \frac{\sqrt{3}}{2}i$**

Lesson 4-4