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LT 4.1 Study Guide and Intervention
Complex Numbers

Pure Imaginary Numbers A **square root** of a number *n* is a number whose square is *n*. For nonnegative real numbers a and b, $\sqrt{ab} = \sqrt{a} \cdot \sqrt{b}$ and $\sqrt{\frac{a}{b}} = \frac{\sqrt{a}}{\sqrt{b}}$, $b \neq 0$.

- The **imaginary unit *i*** is defined to have the property that $\boldsymbol{i}^2 = -1$.
- Simplified square root expressions do not have radicals in the denominator, and any number remaining under the square root has no perfect square factor other than 1.

Example 1

a. Simplify $\sqrt{-48}$.

$\sqrt{-48} = \sqrt{16 \cdot (-3)}$

$= \sqrt{16} \cdot \sqrt{3} \cdot \sqrt{-1}$

$= 4\boldsymbol{i}\sqrt{3}$

b. Simplify $\sqrt{-63}$.

$\sqrt{-63} = \sqrt{-1 \cdot 7 \cdot 9}$

$= \sqrt{-1} \cdot \sqrt{7} \cdot \sqrt{9}$

$= 3\boldsymbol{i}\sqrt{7}$

Example 2

a. Simplify $-3\boldsymbol{i} \cdot 4\boldsymbol{i}$.

$-3\boldsymbol{i} \cdot 4\boldsymbol{i} = -12\boldsymbol{i}^2$

$= -12(-1)$

$= 12$

b. Simplify $\sqrt{-3} \cdot \sqrt{-15}$.

$\sqrt{-3} \cdot \sqrt{-15} = \boldsymbol{i}\sqrt{3} \cdot \boldsymbol{i}\sqrt{15}$

$= \boldsymbol{i}^2\sqrt{45}$

$= \sqrt{-1} \cdot \sqrt{9} \cdot \sqrt{5}$

$= -3\sqrt{5}$

Example 3: Solve $x^2 + 5 = 0$.

$x^2 + 5 = 0$	Original equation.
$x^2 = -5$	Subtract 5 from each side.
$x = \pm\sqrt{5}\boldsymbol{i}$	Square Root Property.

Exercises

Simplify.

1. $\sqrt{-72}$

2. $\sqrt{-24}$

3 . $\sqrt{-84}$

4. $(2 + \boldsymbol{i})(2 - \boldsymbol{i})$

Solve each equation.

5. $5x^2 + 45 = 0$

6. $4x^2 + 24 = 0$

7. $-9x^2 = 9$

8. $7x^2 + 84 = 0$

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LT 4.1 Study Guide and Intervention *(continued)*

Complex Numbers

Operations with Complex Numbers

Complex Number	A complex number is any number that can be written in the form $a + b\boldsymbol{i}$, where a and b are real numbers and $\boldsymbol{i}$ is the imaginary unit ($\boldsymbol{i}^2 = -1$). a is called the real part, and b is called the imaginary part.
Addition and Subtraction of Complex Numbers	Combine like terms. $(a + b\boldsymbol{i}) + (c + d\boldsymbol{i}) = (a + c) + (b + d)\boldsymbol{i}$ $(a + b\boldsymbol{i}) - (c + d\boldsymbol{i}) = (a - c) + (b - d)\boldsymbol{i}$
Multiplication of Complex Numbers	Use the definition of $\boldsymbol{i}^2$ and the FOIL method: $(a + b\boldsymbol{i})(c + d\boldsymbol{i}) = (ac - bd) + (ad + bc)\boldsymbol{i}$
Complex Conjugate	$a + b\boldsymbol{i}$ and $a - b\boldsymbol{i}$ are complex conjugates. The product of complex conjugates is always a real number.

To divide by a complex number, first multiply the dividend and divisor by the **complex conjugate** of the divisor.

Example 1: Simplify $(6 + i) + (4 - 5i)$.

$(6 + \boldsymbol{i}) + (4 - 5\boldsymbol{i})$

$= (6 + 4) + (1 - 5)\boldsymbol{i}$

$= 10 - 4\boldsymbol{i}$

Example 2: Simplify $(8 + 3i) - (6 - 2i)$.

$(8 + 3\boldsymbol{i}) - (6 - 2\boldsymbol{i})$

$= (8 - 6) + [3 - (-2)]\boldsymbol{i}$

$= 2 + 5\boldsymbol{i}$

Example 3: Simplify $(2 - 5i) \cdot (-4 + 2i)$.

$(2 - 5\boldsymbol{i}) \cdot (-4 + 2\boldsymbol{i})$

$= 2(-4) + 2(2\boldsymbol{i}) + (-5\boldsymbol{i})(-4) + (-5\boldsymbol{i})(2\boldsymbol{i})$

$= -8 + 4\boldsymbol{i} + 20\boldsymbol{i} - 10\boldsymbol{i}^2$

$= -8 + 24\boldsymbol{i} - 10(-1)$

$= 2 + 24\boldsymbol{i}$

Example 4: Simplify $\frac{3 - i}{2 + 3i}$.

$\frac{3 - i}{2 + 3i} = \frac{3 - i}{2 + 3i} \cdot \frac{2 - 3i}{2 - 3i}$

$= \frac{6 - - 9i - -2i + 3i^2}{4 - 9i^2}$

$= \frac{3 - 11i}{13}$

$= \frac{3}{13} - \frac{11}{13}i$

Exercises

Simplify.

1. $(-4 + 2i) + (6 - 3i)$

2. $(5 - i) - (3 - 2i)$

3. $(6 - 3i) + (4 - 2i)$

4. $(-11 + 4i) - (1 - 5i)$

5. $(8 + 4i) + (8 - 4i)$

6. $(5 + 2i) - (-6 - 3i)$

7. $(2 + i)(3 - i)$

8. $(5 - 2i)(4 - i)$

9. $(4 - 2i)(1 - 2i)$

10. $\frac{5}{3 + i}$

11. $\frac{7 - 13i}{2i}$

12. $\frac{6 - 5i}{3i}$