

4-4 Study Guide and Intervention

Complex Numbers

Pure Imaginary Numbers A square root of a number n is a number whose square is n . For nonnegative real numbers a and b , $\sqrt{ab} = \sqrt{a} \cdot \sqrt{b}$ and $\sqrt{\frac{a}{b}} = \frac{\sqrt{a}}{\sqrt{b}}$, $b \neq 0$.

- The **imaginary unit** i is defined to have the property that $i^2 = -1$.
- Simplified square root expressions do not have radicals in the denominator, and any number remaining under the square root has no perfect square factor other than 1.

Example 1

a. Simplify $\sqrt{-48}$.

$$\begin{aligned}\sqrt{-48} &= \sqrt{16 \cdot (-3)} \\ &= \sqrt{16} \cdot \sqrt{3} \cdot \sqrt{-1} \\ &= 4i\sqrt{3}\end{aligned}$$

b. Simplify $\sqrt{-63}$.

$$\begin{aligned}\sqrt{-63} &= \sqrt{-1 \cdot 7 \cdot 9} \\ &= \sqrt{-1} \cdot \sqrt{7} \cdot \sqrt{9} \\ &= 3i\sqrt{7}\end{aligned}$$

Example 2

a. Simplify $-3i \cdot 4i$.

$$\begin{aligned}-3i \cdot 4i &= -12i^2 \\ &= -12(-1) \\ &= 12\end{aligned}$$

b. Simplify $\sqrt{-3} \cdot \sqrt{-15}$.

$$\begin{aligned}\sqrt{-3} \cdot \sqrt{-15} &= i\sqrt{3} \cdot i\sqrt{15} \\ &= i^2\sqrt{45} \\ &= -1 \cdot \sqrt{9} \cdot \sqrt{5} \\ &= -3\sqrt{5}\end{aligned}$$

Example 3 Solve $x^2 + 5 = 0$.

$x^2 + 5 = 0$	Original equation.
$x^2 = -5$	Subtract 5 from each side.
$x = \pm\sqrt{5}i$	Square Root Property.

Exercises

Simplify.

1. $\sqrt{-72}$ $6i\sqrt{2}$

2. $\sqrt{-24}$ $2i\sqrt{6}$

3. $\sqrt{-84}$ $2i\sqrt{21}$

4. $(2 + i)(2 - i)$ 5

Solve each equation.

5. $5x^2 + 45 = 0$ $\pm 3i$

6. $4x^2 + 24 = 0$ $\pm i\sqrt{6}$

7. $-9x^2 = 9$ $\pm i$

8. $7x^2 + 84 = 0$ $\pm 2i\sqrt{3}$

4-4 Study Guide and Intervention *(continued)***Complex Numbers****Operations with Complex Numbers**

Complex Number	A complex number is any number that can be written in the form $a + bi$, where a and b are real numbers and i is the imaginary unit ($i^2 = -1$). a is called the real part, and b is called the imaginary part.
Addition and Subtraction of Complex Numbers	Combine like terms. $(a + bi) + (c + di) = (a + c) + (b + d)i$ $(a + bi) - (c + di) = (a - c) + (b - d)i$
Multiplication of Complex Numbers	Use the definition of i^2 and the FOIL method: $(a + bi)(c + di) = (ac - bd) + (ad + bc)i$
Complex Conjugate	$a + bi$ and $a - bi$ are complex conjugates. The product of complex conjugates is always a real number.

To divide by a complex number, first multiply the dividend and divisor by the **complex conjugate** of the divisor.

Example 1 Simplify $(6 + i) + (4 - 5i)$.

$$\begin{aligned}(6 + i) + (4 - 5i) \\ &= (6 + 4) + (1 - 5)i \\ &= 10 - 4i\end{aligned}$$

Example 3 Simplify $(2 - 5i) \cdot (-4 + 2i)$.

$$\begin{aligned}(2 - 5i) \cdot (-4 + 2i) \\ &= 2(-4) + 2(2i) + (-5i)(-4) + (-5i)(2i) \\ &= -8 + 4i + 20i - 10i^2 \\ &= -8 + 24i - 10(-1) \\ &= 2 + 24i\end{aligned}$$

Example 2 Simplify $(8 + 3i) - (6 - 2i)$.

$$\begin{aligned}(8 + 3i) - (6 - 2i) \\ &= (8 - 6) + [3 - (-2)]i \\ &= 2 + 5i\end{aligned}$$

Example 4 Simplify $\frac{3 - i}{2 + 3i}$.

$$\begin{aligned}\frac{3 - i}{2 + 3i} &= \frac{3 - i}{2 + 3i} \cdot \frac{2 - 3i}{2 - 3i} \\ &= \frac{6 - 9i - 2i + 3i^2}{4 - 9i^2} \\ &= \frac{3 - 11i}{13} \\ &= \frac{3}{13} - \frac{11}{13}i\end{aligned}$$

Exercises

Simplify.

- $(-4 + 2i) + (6 - 3i)$
 $2 - i$
- $(5 - i) - (3 - 2i)$
 $2 + i$
- $(6 - 3i) + (4 - 2i)$
 $10 - 5i$
- $(-11 + 4i) - (1 - 5i)$
 $-12 + 9i$
- $(8 + 4i) + (8 - 4i)$
 16
- $(5 + 2i) - (-6 - 3i)$
 $11 + 5i$
- $(2 + i)(3 - i)$ $7 + i$
- $(5 - 2i)(4 - i)$ $18 - 13i$
- $(4 - 2i)(1 - 2i)$ $-10i$
- $\frac{5}{3 + i} - \frac{3}{2} - \frac{1}{2}i$
- $\frac{7 - 13i}{2i} - \frac{13}{2} - \frac{7}{2}i$
- $\frac{6 - 5i}{3i} - \frac{5}{3} - 2i$